MA612L-Partial Differential Equations

Lecture 33: Nonlinear PDE

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Nonlinear PDE-Complete Integral

Important PDEs

So far, we have seen

1. Transport Equation Solution $u_t + b.Du = h$
2. Wave Equation $u_{tt} + \Delta u = h$
3. Laplace/Poisson Equation $\Delta u = h$
4. Heat Equation $u_t + \Delta u = h$
5. Canonical Form and Laplace equation in Polar, Cylindrical, and Spherical Coordinates

Now, let us focus on first-order nonlinear PDEs

First-Order Nonlinear PDE

The general first-order nonlinear PDEs are of the form

$$F(Du, u, \mathbf{x}) = 0$$

where $\mathbf{x} \in \Omega$ and Ω is open subset of $\mathbb{R}^n$. Here

$$F : \mathbb{R}^n \times \mathbb{R} \times \overline{\Omega} \to \mathbb{R}$$

is given and $u : \overline{\Omega} \to \mathbb{R}$ is the unknown. $u = u(\mathbf{x})$.
Let us write,

$$F = F(\mathbf{p}, z, \mathbf{x}) = F(p_1, p_2, \cdots, p_n, z, x_1, x_2, \cdots, x_n)$$

for $\mathbf{p} \in \mathbb{R}^n, z \in \mathbb{R}, \mathbf{x} \in \Omega$. Here, $\mathbf{p} = Du(\mathbf{x}), z = u(\mathbf{x})$. Let us assume that F is smooth from here on.

First-Order Nonlinear PDE

Also, set

$$\begin{cases} D_{\mathbf{p}}F = (F_{p_1}, F_{p_2}, \cdots, F_{p_n}) \\ D_z F = F_z \\ D_x F = (F_{x_1}, F_{x_2}, \cdots, F_{x_n}) \end{cases}$$

Let us find solutions u of the PDE $F(Du, u, \mathbf{x}) = 0$ in Ω subject to the boundary conditions

$$u = f \quad \text{on} \quad \Gamma$$

where Γ is some given subset of $\partial\Omega$ and $g : \Gamma \to \mathbb{R}$ is prescribed.

Complete Integrals

$$F(Du, u, \mathbf{x}) = 0 \tag{1}$$

Suppose $A \subset \mathbb{R}^n$ is an open set. Assume for each parameter $\mathbf{a} = (a_1, a_2, \cdots, a_n) \in A$, we have a C^2 solution $u = u(\mathbf{x}; \mathbf{a})$ of the PDE (1). Now, we write

$$(D_{\mathbf{a}}u, D^2_{\mathbf{xa}}u) = \begin{pmatrix} u_{a_1} & u_{x_1a_1} & \cdots & u_{x_na_1} \\ u_{a_2} & u_{x_1a_2} & \cdots & u_{x_na_2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{a_n} & u_{x_1a_n} & \cdots & u_{x_na_n} \end{pmatrix}_{n\times(n+1)} \tag{2}$$

Complete Integrals

Definition 1 (Complete Integral)
A C^2 function $u = u(\mathbf{x}; \mathbf{a})$ is called a complete integral in $\Omega \times A$ provided

1. $u(\mathbf{x}; \mathbf{a})$ solves the PDE (1) for each $\mathbf{a} \in A$.
2. $\text{rank}(D_{\mathbf{a}}u, D^2_{\mathbf{xa}}u) = n \quad (\mathbf{x} \in \Omega, \mathbf{a} \in A)$

Remarks

- In short, a solution of (1) in which the number of arbitrary constants is equal to the number of independent variables is known as a complete integral or complete solution.
- That is, conditions (1) and (2) in definition 1 ensure $u(\mathbf{x}; \mathbf{a})$ depends on all the n independent parameters $a_1, a_2, \cdots, a_n$.

Complete Integrals

Let us claim the remark as follows: Suppose $B \subset \mathbb{R}^{n-1}$ is open and for each $b \in B$, assume $v = v(\mathbf{x}; \mathbf{b}), \mathbf{x} \in \Omega$ is a solution of (1). Suppose also that there exists a C^1 mapping

$$\psi : A \to B, \psi = (\psi^1, \psi^2, \cdots, \psi^{n-1})$$

such that

$$u(\mathbf{x}; \mathbf{a}) = v(\mathbf{x}; \psi(\mathbf{a})) \;\; (\mathbf{x} \in \Omega, \mathbf{a} \in A) \tag{3}$$

That is, suppose there exists a function $u(\mathbf{x}; \mathbf{a})$ that depends on $n-1$ parameters $b_1, b_2, \cdots, b_{n-1}$. Then

$$u_{x_i a_j}(\mathbf{x}; \mathbf{a}) = \sum_{k=1}^{n-1} v_{x_i b_k}(\mathbf{x}; \psi(a)) \psi^k_{a_j}(a) \quad (i, j = 1, 2, \cdots, n) \tag{4}$$

Complete Integrals

Consequently,

$$det(D^2_{\mathbf{xa}}u) = \sum_{k_1,k_2,\cdots,k_n=1}^{n-1} v_{x_1 b_{k_1}} v_{x_2 b_{k_2}} \cdots v_{x_n b_{k_n}} \begin{pmatrix} \psi^{k_1}_{a_1} & \psi^{k_1}_{a_2} & \cdots & \psi^{k_1}_{a_n} \\ \psi^{k_2}_{a_1} & \psi^{k_2}_{a_2} & \cdots & \psi^{k_2}_{a_n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi^{k_n}_{a_1} & \psi^{k_n}_{a_2} & \cdots & \psi^{k_n}_{a_n} \end{pmatrix} = 0 \quad (5)$$

Reason: For each choice of $k_1, k_2, \cdots, k_n \in \{1, 2, \cdots, n-1\}$, at least two columns in the corresponding matrix are equal. Also,

$$u_{a_j}(\mathbf{x}; \mathbf{a}) = \sum_{k=1}^{n-1} v_{b_k}(\mathbf{x}; \psi(a))\psi^k_{aj}(a) \quad (j = 1, 2, \cdots, n) \quad (6)$$

A similar argument will show that $(D_{\mathbf{a}}u, D^2_{\mathbf{xa}}u) = 0$ and hence rank$(D_{\mathbf{a}}u, D^2_{\mathbf{xa}}u) < n$. Hence the claim.

Complete Integrals

Examples 1

The Clairaut's equation from differential geometry is the PDE

$$\mathbf{x}.Du + f(Du) = u$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is given. Then, its complete integral is given by

$$u(\mathbf{x};\mathbf{a}) = \mathbf{a}.\mathbf{x} + f(\mathbf{a}) \quad (\mathbf{x} \in \Omega, \mathbf{a} \in \mathbb{R}^n)$$

For

$$Du = \mathbf{a} \implies u = \mathbf{x}.Du + f(Du)$$

But how did we get this?

Complete Integrals

The Clairaut's equation from differential geometry is the PDE

$$\mathbf{x}.Du + f(Du) = u$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is given. Let

$$p = Du = \nabla u(\mathbf{x}) \in \mathbb{R}^n.$$

Then the PDE becomes

$$u = \mathbf{x} \cdot p + f(p).$$

Complete Integrals

Differentiating both sides gives

$$u_{x_i} = p_i = \frac{\partial}{\partial x_i}\big(\mathbf{x} \cdot p + f(p)\big).$$

Compute the right-hand side:

$$\frac{\partial}{\partial x_i}(\mathbf{x} \cdot p) = p_i + \sum_{j=1}^{n} x_j \frac{\partial p_j}{\partial x_i},$$

and

$$\frac{\partial}{\partial x_i} f(p) = \sum_{j=1}^{n} f_{p_j}(p)\, \frac{\partial p_j}{\partial x_i}.$$

Thus,

$$p_i = p_i + \sum_{j=1}^{n} \big(x_j + f_{p_j}(p)\big) \frac{\partial p_j}{\partial x_i}.$$

Complete Integrals

After cancelling p_i from both sides, we obtain

$$\sum_{j=1}^{n} (x_j + f_{p_j}(p)) \frac{\partial p_j}{\partial x_i} = 0, \qquad i = 1, \ldots, n.$$

Two cases:

- Case 1: p is constant
- Case 2: p not constant

Complete Integrals

Case 1: p is constant
If $\nabla p = 0$, then $p = \mathbf{a}$ is a constant vector. Integrating $Du = \mathbf{a}$ gives

$$u(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + C.$$

Substituting into the original PDE:

$$\mathbf{x} \cdot \mathbf{a} + f(\mathbf{a}) = \mathbf{a} \cdot \mathbf{x} + C \quad \Rightarrow \quad C = f(\mathbf{a}).$$

Hence the **complete integral** is

$$\boxed{u(\mathbf{x}; \mathbf{a}) = \mathbf{a} \cdot \mathbf{x} + f(\mathbf{a})}, \qquad \mathbf{a} \in \mathbb{R}^n.$$

Complete Integrals

MA612L PARTIAL DIFFERENTIAL EQUATIONS

Examples 2

The eikonal equation from geometric optics is the PDE

$$|Du| = 1$$

Then, its complete integral is given by

$$u(\mathbf{x}; \mathbf{a}, b) = \mathbf{a}.\mathbf{x} + b \quad (\mathbf{x} \in \Omega, \mathbf{a} \in \partial B(\mathbf{0}, 1), b \in \mathbb{R})$$

Again, how?

Question

If

$$|Du| \leq r,$$

Where will be $\mathbf{a}$?

Complete Integrals

Examples 3

The eikonal equation from geometric optics is the PDE

$$|Du| = 1$$

Then, its complete integral is given by

$$u(\mathbf{x};\mathbf{a},b) = \mathbf{a}.\mathbf{x} + b \quad (\mathbf{x} \in \Omega, \mathbf{a} \in \partial B(\mathbf{0},1), b \in \mathbb{R})$$

Again, how?

Complete Integrals

Set $p(\mathbf{x}) := Du(\mathbf{x})$. Then the PDE is the algebraic constraint

$$|p(\mathbf{x})| = 1 \qquad \text{for all } \mathbf{x} \in \Omega.$$

Suppose $p(\mathbf{x})$ is constant on Ω, say $p(\mathbf{x}) \equiv \mathbf{a}$. The PDE forces

$$|\mathbf{a}| = 1,$$

so $\mathbf{a} \in \partial B(\mathbf{0}, 1) = S^{n-1}$. Integrating $Du = \mathbf{a}$ yields

$$u(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + C$$

for some constant $C \in \mathbb{R}$.

Complete Integrals

Substituting into the PDE does not impose any further restriction on C, so the family

$$u(\mathbf{x};\mathbf{a},b) = \mathbf{a}\cdot\mathbf{x} + b, \qquad \mathbf{a}\in S^{n-1},\ b\in\mathbb{R}$$

is a family of classical solutions. This family is precisely the *complete integral* consisting of all planes with unit normal $\mathbf{a}$. The algebraic form of the PDE $|p| = 1$ suggests treating p as a new unknown. Differentiating the identity $|p(\mathbf{x})|^2 = 1$ with respect to x_j gives

$$2\,p(\mathbf{x})\cdot\partial_{x_j}p(\mathbf{x}) = 0, \qquad j = 1,\dots,n,$$

or equivalently

$$p(\mathbf{x})\cdot\partial_{x_j}p(\mathbf{x}) = 0 \quad \text{for each } j.$$

Complete Integrals

Thus each partial derivative $\partial_{x_j} p$ is orthogonal to p. One immediate possibility consistent with these relations is

$$\partial_{x_j} p(\mathbf{x}) \equiv 0 \quad \text{for all } j,$$

i.e. p is constant. This yields the plane solutions above. Hence, the plane family arises naturally as the constant-gradient branch of the PDE, not as a random guess.

Complete Integrals

Examples 4

The Hamilton-Jacobi equation from mechanics, in its simplest form a PDE

$$u_t + H(Du) = 0$$

where $H : \mathbb{R}^n \to \mathbb{R}$. Then, its complete integral is given by

$$u(\mathbf{x}, t; \mathbf{a}, b) = \mathbf{a}.\mathbf{x} + tH(\mathbf{a}) + b \quad (\mathbf{x}, \mathbf{a} \in \mathbb{R}^n, t \geq 0, b \in \mathbb{R})$$

Again, how?

Then the PDE can be read as an algebraic relation

$$u_t + H(p) = 0.$$

Complete Integrals

Assume $p(\mathbf{x}, t) \equiv \mathbf{a}$ is constant (the simplest branch). Then

$$Du = \mathbf{a}, \qquad |\mathbf{a}| \text{ arbitrary},$$

so integrating in $\mathbf{x}$ we get

$$u(\mathbf{x}, t) = \mathbf{a} \cdot \mathbf{x} + \phi(t)$$

for some function $\phi(t)$. Substitute into the PDE:

$$u_t(\mathbf{x}, t) = \phi'(t), \qquad H(Du) = H(\mathbf{a}),$$

hence

$$\phi'(t) + H(\mathbf{a}) = 0 \quad \Longrightarrow \quad \phi(t) = -t\, H(\mathbf{a}) + b,$$

where $b \in \mathbb{R}$ is an integration constant.

Complete Integrals

The family

$$u(\mathbf{x}, t; \mathbf{a}, b) = \mathbf{a} \cdot \mathbf{x} - t\, H(\mathbf{a}) + b, \qquad \mathbf{a} \in \mathbb{R}^n,\ b \in \mathbb{R}$$

consists of classical solutions and is the *complete integral* corresponding to the constant-gradient branch.

Remarks

- The PDE constrains only u_t and Du via an algebraic relation. Taking Du constant is the natural first branch to consider, and integrating with respect to x and t yields the family above.

Particular Integrals

The geometrical interpretation of a complete integral in x, y and u is as follows: It is a relation between x, y and u, that is, the equation of the surface. Since it contains two arbitrary parameters, it belongs to a double infinite system of surfaces or to a single infinite system of a family of surfaces.

Definition 2 (Particular Integral)
If particular values are given to the parameters $\mathbf{a} \in A$, then the solution is called a particular integral or particular solution of (1).

Nonlinear PDE-Envelopes

Envelopes

Let us look at how to build more complicated solutions of (1) which depend on an arbitrary function of $n-1$ variables, but not just on n parameters. Let us construct these new solutions as envelopes of complete integrals. More generally, other $m-$ parameter families of solutions, where $m < n$.

Remarks

1. The envelope v defined in the following definition is sometimes called a *singular integral* of (1).
2. The solution obtained by eliminating the arbitrary constants $\mathbf{a}$ between $F(\mathbf{a}, z\mathbf{x}) = 0$ and $D_{\mathbf{a}}F(\mathbf{a}, z\mathbf{x}) = 0$ is called singular integral or singular solution of (1).

Envelopes

Definition 3 (Envelopes)
Let $u = u(\mathbf{x};\mathbf{a})$ be a C^1 function of $\mathbf{x} \in \Omega, \mathbf{a} \in A$ where $\Omega, A \subset \mathbb{R}^n$ are open sets. Consider the vector equation

$$D_{\mathbf{a}}u(\mathbf{x};\mathbf{a}) = 0 \quad (\mathbf{x} \in \Omega, \mathbf{a} \in A) \tag{7}$$

Suppose we can solve (7) for the parameter $\mathbf{a}$ as a C^1 function of $\mathbf{x}$.

$$\mathbf{a} = \phi(\mathbf{x}) \tag{8}$$

Then

$$D_{\mathbf{a}}u(\mathbf{x};\phi(\mathbf{x})) = 0 \quad (\mathbf{x} \in \Omega) \tag{9}$$

We then call

$$v(\mathbf{x}) := u(\mathbf{x};\phi(\mathbf{x})) = 0 \quad (\mathbf{x} \in \Omega) \tag{10}$$

the envelope of the function $\{u(.;\mathbf{a})\}_{\mathbf{a}\in A}$

Envelopes

Theorem 5 (Construction of New Solutions)
Suppose for each $\mathbf{a} \in A$ as above $u = u(.;\mathbf{a})$ solves (1). Assume further that the envelope v defined by (10) and (9) above exists and is a C^1 function. Then v solves (1) as well.

Proof: From (10), we have

$$\begin{aligned} v_{x_i}(\mathbf{x}) &= u_{x_i}(\mathbf{x};\phi(\mathbf{x})) + \sum_{j=1}^{n} u_{a_j}(\mathbf{x}, \phi(\mathbf{x}))\phi^j_{x_i}(\mathbf{x}) \\ &= u_{x_i}(\mathbf{x};\phi(\mathbf{x})) \end{aligned}$$

The second term vanishes due to (9). Hence for each $\mathbf{x} \in \Omega$, we have

$$F(Dv(\mathbf{x}), v(\mathbf{x}), \mathbf{x}) = F(Du(\mathbf{x};\phi(\mathbf{x})), u(\mathbf{x};\phi(\mathbf{x})), \mathbf{x}) = 0$$

Envelopes: Geometric Interpretation

1. The geometric interpretation for envelope is that for each $\mathbf{x} \in \Omega$, the graph of v is tangent to the graph of $u(.;\mathbf{a})$ for $\mathbf{a} = \phi(\mathbf{x})$. Thus $Dv = D_{\mathbf{x}}u(.;\mathbf{a})$ at $\mathbf{x}$ for $\mathbf{a} = \phi(\mathbf{x})$.
2. The operation of elimination is equivalent to the selection of a representative family from the system of families of surfaces and then finding its envelope.
3. For example, in two dimensions, the above equations represent a curve drawn on the surface of the family whose parameter is a, while the equation obtained by eliminating a between them is the envelope of the family.
4. Hence the envelope touches the surface represented by $F(a, b, z, x, y) = 0$ and $b = \phi(a)$ along the curve represented by $F_a(a, b, z, x, y) = 0$. This curve is called the characteristic of the envelope.

Complete Integrals

Examples 6

The Clairaut's equation from differential geometry is the PDE

$$\mathbf{x}.Du + f(Du) = u$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is given.

Envelopes: Example

Case 2: p is not constant
If $p(\mathbf{x})$ is not constant, the previous condition implies

$$x_j + f_{p_j}(p) = 0, \qquad j = 1, \ldots, n.$$

This relation defines p in terms of $\mathbf{x}$. Substituting into

$$u = \mathbf{x} \cdot p + f(p)$$

then gives the **singular solution**, representing the envelope of the family of planes

$$u(\mathbf{x}; \mathbf{a}) = \mathbf{a} \cdot \mathbf{x} + f(\mathbf{a}).$$

Envelopes: Example

Examples 7

The eikonal equation from geometric optics is the PDE

$$|Du| = 1$$

Envelopes: Example

If $p(\mathbf{x})$ is not constant, the orthogonality conditions restrict how p may vary; geometrically, $p(\mathbf{x})$ can vary only in directions orthogonal to itself. In many geometric constructions, the general solution of $|Du| = 1$ arises as an *envelope* (or supremum/infimum) of the family of planes:

$$u(\mathbf{x}) = \sup_{\mathbf{a} \in S^{n-1}} \big(\mathbf{a} \cdot \mathbf{x} + \phi(\mathbf{a})\big),$$

for some function $\phi : S^{n-1} \to \mathbb{R}$. (Dually, one may use an infimum depending on sign conventions.) The envelope interpretation explains how nonplane solutions (e.g. distance functions) are built from the plane family: at each $\mathbf{x}$ the active parameter $\mathbf{a} = \phi(\mathbf{x})$ is a unit vector giving the tangent plane to the graph of u at $\mathbf{x}$.

Envelopes: Example

Examples 8

The Hamilton-Jacobi equation from mechanics, in its simplest form a PDE

$$u_t + H(Du) = 0$$

where $H : \mathbb{R}^n \to \mathbb{R}$.

Envelopes: Example

Remarks

- For nonconstant $p(\mathbf{x}, t)$ one obtains the full characteristic system

$$\begin{cases} \dot{\mathbf{x}}(s) = H_p(\mathbf{p}(s)), \\ \dot{\mathbf{p}}(s) = 0, \\ \dot{u}(s) = \mathbf{p}(s) \cdot H_p(\mathbf{p}(s)) - H(\mathbf{p}(s)), \end{cases}$$

  so $\mathbf{p}(s)$ is constant along characteristics and $\mathbf{x}(s)$ evolves linearly; integrating yields the same plane–time family locally, and more general solutions arise as envelopes (or viscosity solutions) built from these planes.
- If instead the PDE were $u_t - H(Du) = 0$, replace $-tH(\mathbf{a})$ by $+tH(\mathbf{a})$.

Envelopes: Example

Examples 9

Consider the PDE

$$u^2(1+|Du|^2) = 1$$

A complete integral is

$$u(\mathbf{x}, \mathbf{a}) = \pm\sqrt{1 - |\mathbf{x} - \mathbf{a}|} \quad (|\mathbf{x} - \mathbf{a}| < 1)$$

Now,

$$D_{\mathbf{a}}u = \pm\frac{\mathbf{x} - \mathbf{a}}{\sqrt{1 - |\mathbf{x} - \mathbf{a}|}} = 0$$

provided $\mathbf{a} = \phi(\mathbf{x}) = \mathbf{x}$. Thus, $v \equiv \pm 1$ are singular integrals of the given PDE.

Envelopes: General Integral

To generate still more solutions of (1) from a complete integral, let us vary the above construction. Choose any $A \subset \mathbb{R}^{n-1}$ and any C^1 function $h : A' \to \mathbb{R}$ such that the graph lies within A. Let us write

$$\mathbf{a} = (a_1, a_2, \cdots, a_n) = (\mathbf{a}', a_n), \quad \mathbf{a}' = (a_1, a_2, \cdots, a_{n-1}) \in \mathbb{R}^{n-1}$$

Definition 4 (General Integral)
The general integral (depending on h) is the envelope $v' = v'(\mathbf{x})$ of the functions

$$u'(\mathbf{x}; \mathbf{a}) = u(\mathbf{x}; \mathbf{a}', h(\mathbf{a}')) \quad (\mathbf{x} \in \Omega, \mathbf{a}' \in A')$$

provided this envelope exists in C^1.

Envelopes

Examples 10

An alternative form for a complete integral of the eikonal equation is given as follows for $n = 2$

$$|Du| = 1$$

$$u(\mathbf{x}; \mathbf{a}) = x_1 \cos a_1 + x_2 \sin a_1 + a_2 \quad \mathbf{x}, \mathbf{a} \in \mathbb{R}^2 \tag{11}$$

If we set $h \equiv 0$ such that

$$u'(\mathbf{x}; a_1) = x_1 \cos a_1 + x_2 \sin a_1$$

represents the subfamily of planar solutions of $|Du| = 1$, whose graphs pass through the point $(0, 0, 0) \in \mathbb{R}^3$, then the envelope is computed by solving

$$D_{a_1} u'(\mathbf{x}; a_1) = -x_1 \sin a_1 + x_2 \cos a_1 = 0$$

Envelopes

$$\implies a_1 = \tan^{-1}\left(\frac{x_2}{x_1}\right)$$

Hence,

$$u'(\mathbf{x};a_1) = x_1 \cos\left(\tan^{-1}\left(\frac{x_2}{x_1}\right)\right) + x_2 \sin\left(\tan^{-1}\left(\frac{x_2}{x_1}\right)\right) = \pm|\mathbf{x}|$$

solves $|Du| = 1$ for $\mathbf{x} \neq 0$

General Integral: Remarks

Remarks

1. To compute envelope, we restrict only to parameters $\mathbf{a}$ of the form $a = (\mathbf{a}', h(\mathbf{a}'))$
2. From a complete integral with n arbitrary constants, to construct a general integral, we need an arbitrary function h of $n-1$ variables.

Question

Suppose we find a solution of (1) depending on an arbitrary function h, does it mean we have found all solutions of (1)?

General Integral: Remarks

Answer: No Suppose our PDE is of the following form

$$F(Du, u, \mathbf{x}) = F_1(Du, u, \mathbf{x})F_2(Du, u, \mathbf{x}) = 0$$

Suppose $u_1(\mathbf{x}; \mathbf{a})$ is a complete integral of $F_1(Du, u, \mathbf{x}) = 0$ and we found a general integral corresponding to any function h, we have still missed to find all solutions of the PDE $F_2(Du, u, \mathbf{x}) = 0$.

Question

How do we find these solutions?

Remember Characteristic ODE. Let us have a look at a few more methods to find the complete integral of a nonlinear PDE, namely Charpit's method and Jacobi's method.

Charpit's Method

If the PDE is of the following form in two variables,

$$F(x, y, z, p, q) = 0 \tag{12}$$

then the corresponding characteristic or Charpit's auxiliary equation is given by:

$$\frac{dx}{-F_p} = \frac{dy}{-F_q} = \frac{dz}{-F_p p - F_q q} = \frac{dp}{F_x + F_z p} = \frac{dq}{F_y + F_z q} = \frac{dG}{0} \tag{13}$$

Thanks

Doubts and Suggestions

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