MA621L-Mathematical Modeling

Lecture 2 : Introduction - What? Why? How?

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Model

Definition 1 (Model)
To an observer B, an object A^* is a model of an object A to the extent that B can use A^* to answer questions that interest him about A.

Definition 2 (Best Model)
The best model is the simplest model that still serves its purpose, that is, it is still complex enough to help us understand a system and to solve problems.

Modeling and Simulation Nature

- Definition
  - Problem Definition to be Solved
  - A question to be answered
  - System Definition for which Answer is Required
- System Analysis
  - Identification of parts of the system related to the question
- Modeling
  - Development of a model of the system based on the results of the systems analysis step

Definition 3 (System)
A system is an object or a collection of objects whose properties we want to study

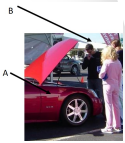
Modeling and Simulation Nature

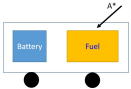
- Simulation
  - Application of the model to the problem
  - Derivation of a strategy to solve the problem
  - Answer the question
- Validation
  - Does the strategy derived in the simulation step solve the problem or answer the question for the real system

Definition 4 (Simulation)
Simulation is the application of a model with the objective of deriving strategies that help to solve a problem or answer a question pertaining to a system.

Model

- System: Car
- System Analysis: Identifying battery and fuel levels
- Modeling:Check battery and fuel level
- Simulation: Model consists of battery and fuel
- Validation: Apply this concept to real car
- Validated: If refilling the tank or battery, starts the car





Steps Involved in Modeling

- System Analysis
  - Use literature, get benefit from others' experiences.
  - Discussions, meetings, and consultations from various disciplines.
  - New data to understand the system and improve the model, and experimental program
- Modeling and Simulation
  - Appropriate software to solve the mathematical model
  - Standard software, customized software, or your own software
- Validation
  - Compare with existing experimental data, from literature or your own experiments, and fit data quantitatively and qualitatively.

Remarks

Quantitative and qualitative fit may fail the validation step if it can't be used to solve the problem.

Conceptual Model and Physical Model

Conceptual Model

- Car drawn on a paper
- No Physical Reality
- All life is problem-solving.
- Conceptual model is an important tool to solve our everyday problems.

Physical Model

- Simplified version of the engine
- Few parts of the engine are connected to the fuel injection process Relating our idea to the real part of the physical world

Mathematical Modelling

Mathematical Model

Mathematics: A Natural Language for Modeling

Conceptual Model and Physical Model

Input Parameters

- Identify a list of input parameters involved in the system

Output Parameters

- Identify the list of Output parameters involved in the system

Input-Output Relation

- Relations between input-output
- Technology

Numerical Data

- Collection of data from experiments, input, output, described naturally in mathematical terms

$$y = f(x)$$

$$(y_1, y_2, \cdots, y_m) = f(x_1, x_2, \cdots, x_n)$$

Initial Study

- The system is a black box
- Uncertainty
- Hypothesis: Experimenter wants to investigate
- Question-And-Answer Game

Mathematical Model

There are different definitions available from different authors related to the mathematical model. To name a few

Definition 5 (Mathematical Model)
A mathematical model is a triplet (S, Q, M) where S is a system,Q is a question relating to S, and M is a set of mathematical statements

$$M = \{\Sigma_1, \Sigma_2, \cdots \Sigma_n\}$$

which can be used to answer Q.

Definition 6 (Mathematical Model)
A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms.

Mathematical Model

A mathematical model consists of the following elements

- Governing Equations
- Supplementary Sub-models: Defining equations and Constitutive equations
- Constraints and Assumptions: Initial and Boundary conditions

Basic Group of Variables

- Decision variables - quantity that the decision-maker controls
- Input variables
- Output variables
- State variables
- Exo/Endogenous variables - in/dependent variable
- Random variables

State Variable

Definition 7 (State Variable)
Let (S, Q, M) be a mathematical model. Mathematical quantities $(s_1, s_2, \cdots, s_n)$ which describe the state of the system S in terms of M and which are required to answer Q are called the state variables of (S, Q, M)

Reduced System

Definition 8 (Reduced System)
Let $s_1, s_2, \cdots, s_n$ be the state variables of a mathematical model (S, Q, M). Let $p_1, p_2, \cdots, p_m$ be mathematical quantities (numbers, variables, functions) which describe properties of the system S in terms of M, and which are needed to compute the state variables. Then $S_r = \{p_1, p_2, \cdots, p_m\}$ is the reduced system and $p_1, p_2, \cdots, p_m$ are the system parameters of (S, Q, M).

Classification of Mathematical Modeling

System

- Physical-Conceptual
- Natural-Technical
- Stochastic-Deterministic
- Continuous-Discrete
- Dimension
- Fields of Application

Classification of Mathematical Modeling

Questions

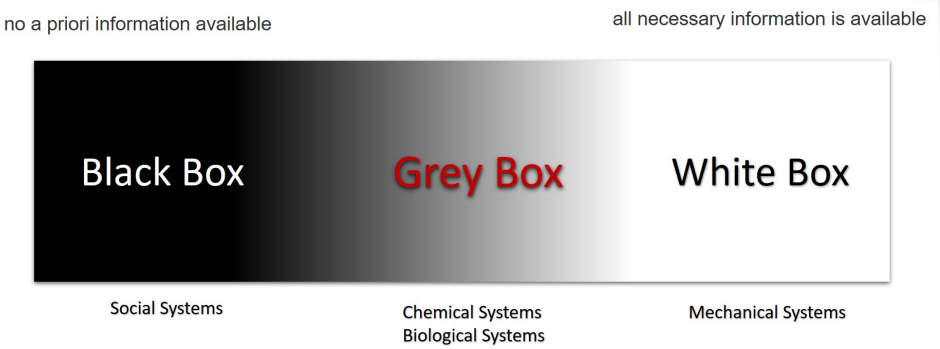
- Phenomenological-Mechanistic
- Stationary-Unstationary
- Lumped-Distributed
- Direct-Inverse
- Research-Management
- Speculation-Design
- Scale

Classification of Mathematical Modeling

Mathematical Statements

- Linear-Nonlinear
- Analytical-Numerical
- Autonomous-Nonautonomous
- Continuous-Discrete
- Difference Equations
- Differential Equations
- Integral Equations
- Algebraic Equations

Black Box to White Box



Don’t

Dont’s

1. Don’t believe that the model is reality
2. Don’t extrapolate the region of fit
3. Don’t distort reality to fit the model
4. Don’t retain a discredited model
5. Don’t fall in love with your model

Summary

In general, the governing equations are written as a functional relationship between input and output parameters. That is

$$\begin{aligned}(y_1, y_2, \cdots, y_m) &= f(x_1, x_2, \cdots, x_n) \\ DV &= f(IV, P, F)\end{aligned}$$

where y_i's and x_j's denote input and output parameters, respectively. DV and IV respectively denote dependent and independent variables. P and F denote parameters and forcing functions, respectively.

Thanks

Doubts and Suggestions

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