MA621L-Mathematical Modeling

Lecture 3 : 10 Questions

Panchatcharam Mariappan[1]

[1]Associate Professor
Department of Mathematics and Statistics
IIT Tirupati, Tirupati

August 17, 2026

Model

Definition 1 (Model)
To an observer B, an object A^* is a model of an object A to the extent that B can use A^* to answer questions that interest him about A.

Definition 2 (Best Model)
The best model is the simplest model that still serves its purpose, that is, it is still complex enough to help us understand a system and to solve problems.

Definition 3 (System)
A system is an object or a collection of objects whose properties we want to study

Modeling and Simulation Nature

Definition 4 (Simulation)
Simulation is the application of a model with the objective of deriving strategies that help to solve a problem or answer a question pertaining to a system.

Definition 5 (Mathematical Model)
A mathematical model is a triplet (S, Q, M) where S is a system, Q is a question relating to S, and M is a set of mathematical statements $M = \{\Sigma_1, \Sigma_2, \cdots \Sigma_n\}$ which can be used to answer Q.

Definition 6 (State Variable)
Let (S, Q, M) be a mathematical model. Mathematical quantities $(s_1, s_2, \cdots, s_n)$ which describe the state of the system S in terms of M and which are required to answer Q are called the state variables of (S, Q, M)

Reduced System

Definition 7 (Reduced System)

Let $s_1, s_2, \cdots, s_n$ be the state variables of a mathematical model (S, Q, M). Let $p_1, p_2, \cdots, p_m$ be mathematical quantities (numbers, variables, functions) which describe properties of the system S in terms of M, and which are needed to compute the state variables. Then $S_r = \{p_1, p_2, \cdots, p_m\}$ is the reduced system and $p_1, p_2, \cdots, p_m$ are the system parameters of (S, Q, M).

Mathematical Modeling: What? Why? How?

What is a Model?

Definition 8 (Mathematical Model)
A **mathematical model** is a representation, in mathematical terms, of the behaviour of real devices and objects.

- Modelling is a *cognitive activity*: we think about how a system behaves and translate that into mathematics.
- A model is always a model *of something, for some purpose*.

Models you already know

$$F = ma, \qquad V = IR, \qquad PV = nRT, \qquad \frac{dN}{dt} = \lambda N$$

What a Model Is NOT

- Not the truth: “all models are wrong, but some are useful”.
- Not unique: sphere $\subset$ ellipsoid $\subset$ geoid all model the Earth.
- Not universal: every model has a **domain of validity** (Ohm’s law fails at high fields; Hooke’s law beyond the elastic limit).
- Not automatically better when complicated: Eratosthenes needed only a sphere.

Why Mathematical Modelling?

Tacoma Narrow Bridge

Figure 1: Tacoma Narrow Bridge

A Story: The Bridge That Galloped

- 1940: the Tacoma Narrows Bridge sways so violently it is nicknamed “Galloping Gertie”.
- Four months after opening, a moderate wind twists the deck and it **collapses**.
- Nobody had modelled the wind–deck interaction. The mathematics existed — it was simply *not asked*.

Why Do We Model?

- **To understand**: why does a phenomenon happen? (science)
    - Newton's Apple Story
- **To predict**: tomorrow / at full scale / at high speed? (forecasting)
- **To design**: will it work *before* we build it? (engineering)
- **To decide and control**: vaccination, traffic signals, interest rates.
- Because experiments may be **impossible** (galaxies), **unethical** (epidemics), **too slow/fast**, or **too expensive**.

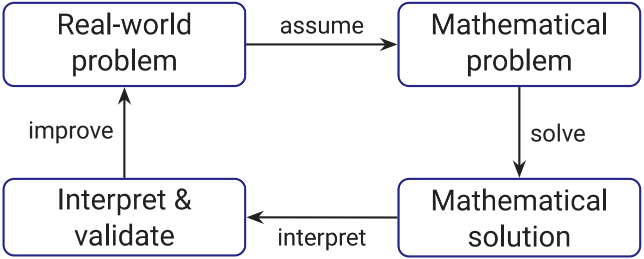
Often the model is the only instrument available.

How Do We Build a Model?

The Ten Principles of Modelling (Dym)

1. **Why?** purpose
2. **Find?** unknowns/outputs
3. **Given?** data
4. **Assume?** idealization
5. **How?** mechanism
6. **Predict?** what it tells us
7. **Valid?** match reality?
8. **Verified?** computations right?
9. **Improve?** refine
10. **Use?** exercise the model

The Modelling Cycle



We iterate until the model is **good enough for its purpose**.

Worked Micro-Example: Coffee on Your Desk

Why? When can I drink it?
Find? $T(t)$.
Given? $T(0) = 90°$C, room $T_m = 25°$C.
Assume? rate $\propto$ excess.

How? $$\frac{dT}{dt} = -k\,(T - T_m), \qquad k > 0$$

Predict? $T(t) = T_m + (T_0 - T_m)e^{-kt}$ — exponential approach to room temperature.
Valid? measure twice, estimate k, test at a third time.
Improve? evaporation, cup geometry — only *if the purpose demands it*.

Thanks

Doubts and Suggestions

panch.m@iittp.ac.in

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