

INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
Department of Mathematics and Statistics

MA621L — Mathematical Modelling

Project-2

Deadline: November 14, 2026

Max. Marks: 9

Instructions: Your report must state the assumptions as A1–A10, each written so that it can be lifted independently, and must identify the triple (S, Q, M) for your model. Marks are awarded for formulation, not for computation. You can use AI tools, but they mostly won't help you for precise answers, as each problem has an experiment and data collection. Also, submit the report in handwritten format, including any figures you draw. Each problem takes 3 marks. Each problem includes an experiment and/or data collection, so you must conduct it and develop the model.

Pacing a 400 m run

A runner wants to cover 400 m in the least possible time. Develop a model in which the velocity obeys $\dot{v} = f - v/\tau$, where f is the propulsive force per unit mass, together with an equation for the available energy. The energy is replenished at a constant rate and used up by the work done. With bounds on f and the condition that the energy cannot become negative, pose the choice of $f(t)$ as an optimal control problem and describe the structure of the optimal strategy. Estimate the parameters from your own 100 m and 400 m times on the campus ground. Compare the predicted speed profile with 50 m split times recorded by friends standing along the track. Discuss which of your assumptions fails first as the race distance grows.

Equipment: stopwatch apps on several phones, and a measuring tape or the track markings.

Mould on a slice of bread

A slice of bread is kept moist in a sealed plastic bag, and mould spreads over it. Photograph the slice each day against squared paper and record the fraction of its area that is covered. Develop a difference-equation model for this fraction, take one day as the time step, and fit its parameters to your data. Find the fixed points and determine their stability, and compare your model with the logistic ODE sampled at daily intervals. Repeat with a second slice kept in a cooler place, such as near a window at night, and report which parameters change. Explain whether daily photographs are frequent enough for a discrete model to be justified. Comment on whether your model can say anything about where the mould appears on the slice, as opposed to how much there is.

Equipment: two slices of bread, zip-lock bags, squared paper, and a phone camera. Keep the bags sealed throughout and discard them unopened.

Draining a water bottle

A plastic water bottle has a small hole made near its base with a pin. Starting from the Navier–Stokes equations, state the reductions that lead to Torricelli’s law and obtain an ODE for the water level $h(t)$. Use it to predict the time taken to drain the bottle. Identify the discharge coefficient from your own measurements of the level against time. Show that the equation $\dot{h} = -k\sqrt{h}$ empties the bottle in finite time. Then show that, given only that the bottle is empty at some instant, its earlier history cannot be recovered, and explain what this says about the well-posedness of the backward problem. Identify the dimensionless group that decides whether viscosity can be neglected. Repeat the experiment with a much smaller hole to find where your model stops working.

Equipment: used water bottles, a pin, a ruler taped to the bottle, and a phone camera.
