INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI

Department of Mathematics and Statistics

MA633L - Numerical Analysis

Problems on Numerical Interpolation

Note: Usual Notations are used. Questions may have some typos or grammatical mistakes as proofreading is not done for this.

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1. Find the polynomials of least degree that interpolate these sets of data (either using Newton's divided difference or Lagrange interpolation)

(a)

x	5	-7	-6	0
y	1	-23	-54	-954

(b)

x	3	7
y	5	-1

(c)

x	7	1	2
y	146	2	1

(d)

x	3	7	1	2
y	10	146	2	1

(e)

x	3	7	1	2
y	12	146	2	1

(f)

x	1.5	2.7	3.1	-2.1	-6.6	11.0
y	0.0	0.0	0.0	1.0	0.0	0.0

(g)

x	1	2	0	3
y	3	2	-4	5

(h)

x	2	0	3
y	11	7	8

2. Find the polynomials of least degree that interpolate these sets of data (either using Newton's divided difference or Lagrange interpolation). Verify that both polynomials are same

(a)

x	-2	0	1
y	0	1	-1

(b)

x	$-\sqrt{\frac{3}{5}}$	0	$\sqrt{\frac{3}{5}}$
y	$f\left(-\sqrt{\frac{3}{5}}\right)$	0	$\left(\sqrt{\frac{3}{5}}\right)$

(c)

x	1	3	2	6
y	-2	-22	-1	-37

(d)

x	3	1	5	6
y	1	-3	2	4

(e)

x	1	3/2	0	2
y	3	13/4	3	5/3

(f)

x	0	1	2	7
y	51	3	1	201

(g)

x	4	2	0	3
y	63	11	7	28

(h)

x	4	2	0	3
y	63	11	7	28

3. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation).

x	-1	0	1	2	3
$f(x)$	2	1	2	-7	10

Verify the polynomial $P_3(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1)$ interpolates the first four points. By adding one additional term to $P_3(x)$, find $P_4(x)$ that interpolates the whole table.

4. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation).

(a)

x	0	1	2
y	2	-4	44

(b)

x	0	-1	1
y	1	2	3

5. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation). Compute $f(1.5)$

x	0	1	2	3	4
$f(x)$	1	5	31	121	341

6. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation). Compute $f(1.2525)$ and $f(1.3325)$

x	1	2	3	4	5	6	7	8	9
$f(x)$	1.5709	1.5713	1.5719	1.5727	1.5738	1.5751	1.5767	1.5785	1.5805

7. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation). Compute $f(2.5)$ and $f(1.25)$ and $f(3.25)$ using $P_3(x)$ and then computing $f(2.5)$ using $P_4(x)$

x	1	1.5	2	3	3.5	4
$f(x)$	0.0	0.17609	0.30103	0.47712	0.54407	0.60206

8. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation). Compute $f(2.05)$, $f(2.15)$ and $f(2.35)$

x	2	2.1	2.2	2.3	2.4
$f(x)$	1.414214	1.449138	1.483240	1.516575	1.549193

9. Compute the third-order Newton's interpolating polynomial for the following data

x	1	4	6	5
$f(x)$	0	1.386294	1.791759	1.609438

If $f(x) = \ln x$, compute $P_3(2)$ and ε_t

10. Use a Lagrante interpolating polynomial of the first and second order to evaluate the density of unused motor oil at $T = 15°C$ based on the following data

x	0	20	40
$T(x)$	3.85	0.800	0.212

11. The following data for the density of nitrogen gas versus temperature come from a table that was measured with high precision. Use first-through fifth-order polynomials to estimate the density at a temperature of 330 K. What is your best estimate? Employ this best estimate and inverse interpolation to determine the corresponding temperature

$T(K)$	200	250	300	350	400	450
Density (kg/m^3)	1.708	1.367	1.139	0.967	0.854	0.759

12. Use Newton's interpolating polynomial to determine y at $x = 3.5$ to the best possible accuracy.

x	0	1	2.5	3	4.5	5	6
y	2	5.4375	7.3516	7.5625	8.4453	9.1875	12

13. Use Newton's interpolating polynomial to determine y at $x = 8$ to the best possible accuracy.

x	0	1	2	5.5	11	13	16	18
y	0.5	3.134	5.3	9.9	10.2	9.35	7.2	6.2

14. Fill out the missing value using Newton's interpolating polynomial or Lagrange interpolation polynomial.

x	1	2	2.4	2.5	3	3.4	4	5
y	0	5	?	6.5	7	?	3	1

15. Given the data

x	1	2	3	5	6
$f(x)$	7	4	5.5	40	82

Calculate $P_1(4), P_2(4), P_3(4)$ and $P_4(4)$ using Newton's divided difference method and Lagrange interpolation method.

16. The following data show the relationship between the viscosity of SAE 70 oil and temperature.

Temperature$T°C$	26.67	93.33	148.89	315.56
viscosity$\mu N.s/m^2$	1.35	0.085	0.012	0.00075

Use quadratic and cubic interpolation to determine the oxygen concentration for $T = 100°C$ (Newton's or Lagrange).

17. The following table gives the dissolved oxygen concentration in waster as a function of temperature ($°C$) and chloride concentration (g/L)

Temperature $T°C$	$c = 0g/L$	$c = 10g/L$	$c = 20g/L$
0	14.6	12.9	11.4
5	12.8	11.3	10.3
10	11.3	10.1	8.96
15	10.1	9.03	8.08
20	9.09	8.17	7.35
25	8.26	7.46	6.73
30	7.56	6.85	6.20

(a) Use Newton's quadratic and cubic interpolation to determine the oxygen concentration for $T = 100°C$ and $c = 10g/L$.

(b) Use Lagrange's linear and quadratic interpolation to determine the oxygen concentration for $T = 100°C$ and $c = 15g/L$.

18. Find the polynomials of least degree that interpolate these sets of data (either using Newton's forward divided difference or backward divided difference or Lagrange interpolation). Compute $J_0(2.05)$, $J_0(2.45)$ and $J_0(2.75)$ using appropriate $P_0(x), P_1(x), P_2(x), \cdots, P_8(x)$

x	$J_0(x)$
2	0.2238907791
2.1	0.1666069803
2.2	0.1103622669
2.3	0.0555397844
2.4	0.0025076832
2.5	-0.0483837764
2.6	-0.0968049544
2.7	-0.1424493700
2.8	-0.1850360334
2.9	-0.2243115458

19. Employ inverse interpolation using a cubic interpolating polynomial and bisection to determine the value of x that corresponds to $f(x) = 1.7$ for the following tabulated data:

x	1	2	3	4	5	6	7
$f(x)$	3.6	1.8	1.2	0.9	0.72	1.5	0.51429

20. Employ inverse interpolation to determine the value of x that corresponds to $f(x) = 0.93$ for the following tabulated data:

x	0	1	2	3	4	5
$f(x)$	0	0.5	0.8	0.9	0.941176	0.961538

Use quadratic interpolation and the quadratic formula to determine the value numerically.

21. Use the portion of the given steam table for super-heated water at 200 MPa to find
    (a) the corresponding entropy s for a specific volume v of 0.118 with linear interpolation,
    (b) the same corresponding entropy using quadratic interpolation, and
    (c) the volume corresponding to an entropy of 6.45 using inverse interpolation

$v, m^3/kg$	0.10377	0.11144	0.12547
$s, kJ/(kgK)$	6.4147	6.5453	6.7664

22. The specific volume of a super heated steam is listed in steam tables for various temperatures at a pressure of $3000 lb/in^2$. Compute v at $T = 750°F$ using Lagrange interpolating polynomial $P_4(x)$

$T°C$	370	382	394	406	418
$v, m^3/kg$	5.9313	7.5838	8.8428	9.796	10.5311

23. The vertical stress σ_z under the corner of a rectangular area subjected to a uniform load of intensity q is given by the solution of Boussinesq's equation

$$\sigma = \frac{q}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\frac{m^2+n^2+2}{m^2+n^2+1}\right] + \sin^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)$$

The value of σ is given as follows for different m and n Use a third order interpolation polynomial to find the value of σ at $m = 0.46$ and $n = 1.4$.

m	$n = 1.2$	$n = 1.4$	$n = 1.6$
0.1	0.02926	0.03007	0.03058
0.2	0.05733	0.05894	0.05994
0.3	0.08323	0.08561	0.08709
0.4	0.10631	0.10941	0.11135
0.5	0.12626	0.13003	0.13241
0.6	0.14309	0.14749	0.15027
0.7	0.15703	0.16199	0.16515
0.8	0.16843	0.17389	0.17739

24. Ohm's law states that the voltage drop V across an ideal resistor is linearly proportional to the current i flowing through the resister as in $V = iR$, where R is the resistance. However, real resistors may not always obey Ohm's law. Suppose that you performed some very precise experiments to measure the voltage drop and corresponding current for a resistor. The following results suggest a curvilinear relationship rather than the straight line represented by Ohm's law. Use a Lagrange interpolating polynomial $P_5(i)$ to fit the data and compute V for $i = 0.10$.

i	-1	-0.5	-0.25	0.25	0.5	1
V	-637	-96.5	-20.5	20.5	96.5	637

25. Bessel functions often arise in advanced engineering analyses such as the study of electric fields. Here are some selected values for the zero-order Bessel function of the first kind. Estimate $J_1(2.1)$ using $P_3(x)$ and $P_4(x)$ using Newton's divided difference

x	1.8	2.0	2.2	2.4	2.6
$J_1(x)$	0.5815	0.5767	0.5560	0.5202	0.4708

26. You measure the voltage drop V across a resistor for a number of different values of current i. The results are

i	0.25	0.75	1.25	1.5	2.0
V	-0.45	-0.6	0.7	1.88	6.0

Compute $P_1(1.15), P_2(1.15), P_3(1.15)$ and $P_4(1.15)$ using Newton's divided difference formula and interpret your results.

27. The current in a wire is measured with great precision as a function of time:

t	0	0.125	0.25	0.3750	0.5
i	0	6.24	7.75	4.85	0.0

Compute $P_1(0.23), P_2(0.23), P_3(0.23)$ and $P_4(0.23)$ using Newton's divided difference formula and interpret your results.

28. The acceleration due to gravity at an altitude y above the surface of the earth is given by

$y(m)$	0	30000	60000	90000	120000
$g(m/s^2)$	9.81	9.7487	9.6879	9.6278	9.5682

Compute g at $y = 55000m$ using Newton's interpolating polynomial $P_4(x)$.

29. Based on an experiment on heated plate, temperatures are measured at various points as given in below table. Estimate the temperature at $(x, y) = (4, 3.2)$ and $(x, y) = (4.3, 2.7)$ using Lagrange interpolating polynomial $P_4(x)$

y	$x = 0$	$x = 2$	$x = 4$	x=6	x=8
0	100	90	80	70	60
2	85	64.49	53.5	48.15	50
4	70	48.9	38.43	35.03	40
6	55	38.78	30.39	27.07	30
8	40	35	30	25	20

30. The domestic oil production of a country from 1990 to 2020 measured in m^3 is given in below table. Estimate the oil production of the country for 2000 using Newton's interpolating polynomial of degree $P_7(x)$

year	1990	1992	1994	1996	1998	2002	2006	2010
Oil m^3	28.528	48.771	94.542	146.282	168.744	173.649	136.577	104.354

31. Find the polynomial of degree as low as possible, interpolating the points (Newton or Lagrange)

x	-1.0	0	0.5	1.0
y	0.25	-0.5	0.25	2.75

32. Find the polynomial of degree as low as possible, interpolating the points (Newton or Lagrange)

x	0	0.6	1.0
y	0	0.75	0.5

33. Use the portion of the given steam table for super heated H_2O at 200 MPa to

(a) find the corresponding entropy s for a specific volume v of $0.108m^3/kg$ with linear interpolation,

(b) find the same corresponding entropy using quadratic interpolation, and

(c) find the volume corresponding to an entropy of 6.6 using inverse interpolation

$v(m^3/kg)$	0.10377	0.11144	0.12540
$s(kJ/kg.K)$	6.4147	6.5453	6.7664

34. The following table gives the distance in nautical kms of the visible horizon for the given heights in feet above the earth's surface. Find the distance of the visible horizon for 48m and 125m using Lagrange interpolation method

$x(height, ft)$	100	150	200	250	300	350	400
$y(distance, km)$	17.10	20.96	24.06	26.8960	29.4720	31.8400	34.0320

35. The following table gives the number of students who obtained marks in the given range. Estimate the number of students who obtained the marks between 40 and 45 using Newton's divided difference method

Marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

36. Find the cubic polynomial that takes the following values. Find the value of $f(4)$ using Newton's forward difference method

x	0	1	2	3
$f(x)$	1	2	1	10

37. Using Newton's backward difference formula, construct an interpolating polynomial of degree 3 for the given data and then find $f(-1/3)$

x	0.75	-0.5	-0.25	0
$f(x)$	-0.0718125	-0.02475	0.3349375	1.10100

38. Using Newton 's forward formula, fin d the value of $f(1.6)$, if

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

39. From the following table find y when $x = 1.85$ and $x = 2.4$ by Newton's interpolation formula

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(x)$	5.474	6.050	6.686	7.389	8.166	9.025	9.974

40. Express the value of θ in terms of x using the following data using Lagrange interpolation:

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

41. Given $\sin(45°) = 0.7071, \sin(50°) = 0.7660, \sin(65°) = 0.8192, \sin(60°) = 0.8660$, find $\sin(52°)$using Newton's forward difference formula.

42. Using the following table and Newton's backward difference formula, estimate $f(0.7)$

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	2.68	3.04	3.38	3.68	3.96	4.21

43. Using the following table and Newton's backward difference formula, estimate $\log(337.5)$ and $\log(327.5)$.

$x°$	310	320	330	340	350	360
$\log(x)$	2.49136	2.50515	2.51851	2.53148	2.54407	2.55630

44. Using the following table and Newton's backward difference formula, estimate $\cos(25°)$ and $\cos(73°)$.

$x°$	10	20	30	40	50	60	70	80
$\cos(x)$	0.9848	0.9397	0.8660	0.7660	0.6428	0.5000	0.3420	0.1737

45. Using the following table and Newton's backward difference formula, estimate $\tan(25°)$ and $\tan(16°)$.

$x°$ 0		10	20	30
$\tan(x)$	0	0.1763	0.3640	0.5774

46. Using the following table and Newton's backward difference formula, estimate $\tan(16°)$.

$x°$	0	5	10	15	20	25	30
$\cos(x)$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

47. A test performed on a NPN transistor gives the following result. Using the Lagrange interpolation, estimate the value of the collector current for the base current of $0.005mA$ and using inverse interpolation, estimate the value of base current required for a collector correct of $4.0mA$.

Base Current (mA)	0	0.01	0.02	0.03	0.04	0.05
Collector Current I_C (mA)	0	1.2	2.5	3.6	4.3	5.34

48. Find $f(22)$ from the following data using Newton's backward difference formula

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

49. Find $f(27.5)$ from the following data using Newton's backward difference formula

x	25	26	27	28	29	30
$f(x)$	4	3.846	3.704	3.571	3.448	3.333

50. Estimate the number of men getting wages between Rs. 10 and 15 (Lakhs) from the following data using Newton's forward difference formula:

Wages in Rs (Lakhs)	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

51. Estimate the number of people getting incomes between Rs. 2000 and 2500 per day in a village from the following data using Newton's forward difference formula:

Income per day	500	500-1000	1000-2000	2000-3000	3000-4000
No. of Persons	6000	4250	3600	1500	650

52. Construct Newton's forward interpolation polynomial for the following data and the estimate the value of y for $x = 5$:

x	4	6	8	10
y	1	3	8	16

53. Find the cubic polynomial using Lagrange interpolation which takes the following values: $y(0) = 1, y(1) = 0, y(2) = 1$ and $y(3) = 10$. Estimate $y(2.5)$.

54. Find the cubic polynomial using Lagrange interpolation which takes the following values: $y(20) = 2854, y(28) = 3162, y(36) = 7088$ and $y(44) = 7984$. Estimate $y(30)$.

55. The following table gives the population of a town during the last six censuses. Using Newton's divided difference formula, estimate the increase in the population during the period from 1939 to 1989:

Year	1939	1949	1959	1969	1979	1989
Population (in 1000's)	12	15	20	27	39	52

56. Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials

    (a) $f(-1/3)$ if $f(-3/4) = -0.07181250, f(-1/2) = -0.02475, f(-1/4) = 0.33493750, f(0) = 1.101$

    (b) $f(1/4)$ if $f(1/10) = -0.62049958, f(1/5) = -0.28398668, f(3/10) = 0.00660095, f(2/5) = 0.2484244$

57. The following table gives the population of a town during the last six censuses. Using Newton's divided difference formula, estimate the increase in the population during the period from 1976 to 1978:

Year	1941	1951	1961	1971	1981	1991
Population (in 1000's)	12	15	20	27	39	52

58. During a data collection regarding export of a certain commodity during five years, customs is unable to find the record for the year 1991. How could you help the customs to estimate the export of the commodity during 1991.

Year	1989	1990	1991	1992	1993
Export (in tons)	443	384	-	397	467

59. When you record a series of data, you found the following sequence. $y_{21} = 18.4708, y_{25} = 17.8144, y_{29} = 17.1070, y_{33} = 16.3432$ and $y_{37} = 15.5154$. Find the value of y_{30} in this sequence.

60. Find the value of y_{25} from the following data $y_{20} = 2854, y_{24} = 3162, y_{28} = 3544, y_{32} = 3992$.

61. Find the $y(25)$, given that $y(20) = 24, y(24) = 32, y(28) = 35, y(32) = 40$, using Newton's forward difference formula.

62. Find the distance moved by a particle and its acceleration at the end of 4 seconds, if the time verses velocity data is as follows:

t	0	1	3	4
v	21	15	12	10

63. Find the missing term in the following table using Newton's divided difference interpolation:

x	0	1	2	3	4
y	1	3	9	-	81

64. The following are the measurements T made on a curve recorded by oscillograph representing a change of current I due to a change in the conditions of an electric current.

T	1.2	2.0	2.5	3.0
I	1.36	0.58	0.34	0.2

Find I when $T = 1.6$

65. Approximate $f(0.05)$ using the following data and the Newton forward-difference formula

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1	1.22140	1.49182	1.82212	2.22554

66. For the five data points $(0,8),(1,12),(3,2),(4,6),(8,0)$, construct a the linear spline and quadratic spline.

67. Find a quadratic spline interpolant for these data

x	-1	0	0.5	1	2	2.5
y	2	1	0	1	2	3

68. Are these functions quadratic splines? Why or Why not?

(a) $Q(x) = \begin{cases} 0.1x^2 & x \in [0,1] \\ 9.3x^2 - 18.4x + 9.1 & x \in [1, 1.3] \end{cases}$

(b) $Q(x) = \begin{cases} -x^2 & x \in [-100, 0] \\ x & x \in [0, 100] \end{cases}$

(c) $Q(x) = \begin{cases} x & x \in [-50, 1] \\ x^2 & x \in [1, 2] \\ 4 & x \in [2, 50] \end{cases}$

69. Is $S(x) = |x|$ a first degree spline? Why or Why not?

70. Derive the equations of the natural cubic spline for the following table

(a)

x	-1	0	1
y	1	2	-1

(b)

x	1	2	3	4	2
y	0	1	0	1	0

71. Do there exist a, b, c and d such that the function

$$S(x) = \begin{cases} ax^3 + x^2 + cx & x \in [-1, 0] \\ bx^3 + x^2 + dx & x \in [0, 1] \end{cases}$$

is a natural cubic spline function that agrees with $|x|$ at the knots $-1, 0, 1$? Justify your answer.

72. Do there exist a, b, c and d such that the function

(a) $S(x) = \begin{cases} -x & x \in [-10, -1] \\ ax^3 + bx^2 + cx + d & x \in [-1, 1] \\ x & x \in [1, 10] \end{cases}$

(b) $S(x) = \begin{cases} x + 1 & x \in [-2, -1] \\ ax^3 + bx^2 + cx + d & x \in [-1, 1] \\ x - 1 & x \in [1, 2] \end{cases}$

(c) $S(x) = \begin{cases} x^3 - 1 & x \in [-9, 0] \\ ax^3 + bx^2 + cx + d & x \in [0, 5] \end{cases}$

(d) $S(x) = \begin{cases} x^2 + x^3 & x \in [0, 1] \\ a + bx + cx^2 + dx^3 & x \in [1, 2] \end{cases}$

is a natural cubic spline function?Justify your answer.

73. Do there exist a, b, c, d and e such that the function

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3 & x \in [0, 1] \\ (x-1)^3 + ex^2 - 1 & x \in [1, 2] \end{cases}$$

is a natural cubic spline function?Justify your answer.

74. Check whether the following is a cubic spline or not

(a) $S(x) = \begin{cases} 1 + 2(x+1) + (x+1)^3 & x \in [-1, 0] \\ 3 + 5x + 3x^2 & x \in [0, 1] \\ 11 + (x-1) + 3(x-1)^2 + (x-1)^3 & x \in [1, 2] \end{cases}$

(b) $S(x) = \begin{cases} x + 1 & x \in [-2, -1] \\ x^3 - 2x + 1 & x \in [-1, 1] \\ x - 1 & x \in [1, 2] \end{cases}$

(c) $S(x) = \begin{cases} x^3 + x - 1 & x \in [-1, 0] \\ x^3 - x - 1 & x \in [0, 1] \end{cases}$

(d) $S(x) = \begin{cases} x^3 + 3x^2 + 7x - 5 & x \in [-1, 0] \\ -x^3 + 3x^2 + 7x - 5 & x \in [0, 1] \end{cases}$

(e) $S(x) = \begin{cases} x^3 + x - 1 & x \in [0, 1] \\ -(x-1)^3 + 3(x-1)^2 + 4(x-1) + 1 & x \in [1, 2] \end{cases}$

(f) $S(x) = \begin{cases} 28 + 25x + 9x^2 + x^3 & x \in [-3, -1] \\ 26 + 19x + 3x^2 - x^3 & x \in [-1, 0] \\ 26 + 19x + 3x^2 - 2x^3 & x \in [0, 3] \\ -163 + 208x - 60x^2 + 5x^3 & x \in [3, 4] \end{cases}$

75. Determine the natural cubic spline that interpolates $f(x) = x^6$ over the interval $[0, 1]$ using knots $0, 1$ and 2

76. Use Hermite polynomial that agrees with the following data and find the missing value t

(a)

x	$f(x)$	$f'(x)$
1.3	0.6200860	−0.5220232
1.5	t	
1.6	0.4554022	−0.5698959
1.9	0.2818186	−0.5811571

(b)

x	$f(x)$	$f'(x)$
8.3	17.56492	3.116256
8.5	t	
8.6	18.50515	3.151762

(c)

x	$f(x)$	$f'(x)$
0.8	0.22363362	2.1691753
0.9	t	
1.0	0.65809197	2.0466965

(d)

x	$f(x)$	$f'(x)$
−0.5	−0.02475	0.751
−0.25	0.3349375	2.189
−0.25	t	
0	1.101	4.002

77. If we interpolate the function $f(x) = e^{x-1}$ with a polynomial $P_{12}(x)$ using 13 nodes in $[-1, 1]$, what is a good upper bound for $|f(x) - P_{12}(x)|$ on $[-1, 1]$

78. Prove that if f is a polynomial of degree 3, then $f[x_0, x_1, x_2, x_3, x_4] = 0$. Can we generalize this to the following statement:

    If f is a polynomial of degree k, then for $n > k$, $f[x_0, x_1, x_2, \cdots, x_n] = 0$. If so, prove it, if not, give an example.

79. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0), (0.5, y), (1, 3)$ and $(2, 2)$. Find y if the coefficient of x^3 in $P_3(x)$ is 6

80. Let $f(x) = \sqrt{x - x^2}$ and $P_2(x)$ be the interpolation polynomial on $x_0 = 0, x_1$ and $x_2 = 1$. Find the largest value of x_1 in $(0, 1)$ for which $f(0.5) - P_2(0.5) = -0.25$

81. Use the Lagrange interpolating polynomial of degree three or less and four-digit chopping arithmetic to approximate cos(0.750) using the following tables.

x°	0.698	0.733	0.768	0.803
$\cos(x)$	0.7661	0.7432	0.7193	0.6946

    The actual value of $\cos(0.750) = 0.7317$ (to four decimal places). Explain the discrepancy between the actual error and the error bound.

82. If you have solved all above problems, you can observe that both Lagrange interpolation and Newton's divided difference polynomials are matching.

(a) Prove that

$$l_0(x)f_0 + l_1(x)f_1 = f[x_0] + f[x_0, x_1](x - x_0)$$

(b) Prove that

$$l_0(x)f_0 + l_1(x)f_1 + l_2(x)f_2 = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

(c) Can we generalize this to the following statement?

For any nodes $x_0, x_1, \cdots, x_n$ and for any f

$$\sum_{i=0}^{n} f(x_i)l_i(x) = \sum_{k=0}^{n} f[x_0, x_1, \cdots, x_k] \prod_{j=0}^{k-1}(x - x_j)$$

If so, prove it, if not, give an example.

83. Prove the following

(a)

$$f[x_0, x_1] = \sum_{i=0}^{1} \frac{f(x_i)}{w'(x_i)}$$

where $w(x) = \prod_{i=0}^{1}(x - x_i)$.

(b)

$$f[x_0, x_1, x_2] = \sum_{i=0}^{2} \frac{f(x_i)}{w'(x_i)}$$

where $w(x) = \prod_{i=0}^{2}(x - x_i)$.

(c) Can we generalize this to the following statement:

$$f[x_0, x_1, \cdots, x_n] = \sum_{i=0}^{n} \frac{f(x_i)}{w'(x_i)}$$

where $w(x) = \prod_{i=0}^{n}(x - x_i)$. If so, prove it, if not, give a counter example.

84. Let $f(x) = 1/x$, then prove that

$$f[x_0, x_1, \cdots, x_n] = (-1)^n \prod_{i=0}^{n} \frac{1}{x_i}$$

85. A table of values of $\cos x$ is required so that linear interpolation will give six-decimal place accuracy for any value of $x \in [0, \pi]$. If the tabular values are to be equally spaced, what is the minimum number of entries needed in the table?

86. The function defined by
$$f(x) = \int_0^x \sin t^2 dt$$
has been tabulated for equally spaced values of x with step $h = 0.1$. What is the maximum error encountered if $P_3(x)$ is to be used to calculate $f(x)$ for any point $x \in [0, \pi/2]$.

87. Using interpolation error theorems, compute the lower bound on the interpolation error $|f(\overline{x}) - P_n(\overline{x})|$ when $f(x) = \ln x, n = 3, x_0 = 1, x_1 = 4/3, x_2 = 5/3, x_3 = 2$ and $\overline{x} = 3/2$.

88. A fourth degree polynomial $P(x) = P_4(x)$ satisfies $\Delta^4 P(0) = 24, \Delta^3 P(0) = 6$ and $\Delta^2 P(0) = 0$. Compute $\Delta^2 P(10)$.

89. The following table are taken from a polynomial of degree ≤ 5. What is the degree of the polynomial

x	-2	-1	0	1	2	3
$f(x)$	-5	1	1	1	7	25

90. How many points do you need to construct a cubic interpolation?

91. Check that the polynomials
$$P_3(x) = 5x^3 - 27x^2 + 45x - 21$$
$$Q_4(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$
both interpolate the points given in the below table

x	1	2	3	4
$f(x)$	2	1	6	47

Does it contradict the uniqueness property? Justify your answer.

92. Show that both polynomials
$$P_3(x) = 3 - 2(x+1) + 0(x+1)x + (x+1)x(x-1)$$
$$Q_3(x) = -1 + 4(x+2) - 3(x+2)(x+1) + (x+2)(x+1)x$$
interpolate the data given in the below table

x	-2	-1	0	1	2
$f(x)$	-1	3	1	-1	3

Does it contradict the uniqueness property? Justify your answer.

93. The following data are given for a polynomial $P(x)$ of unknown degree

x	0	1	2
$P(x)$	2	-1	4

Determine the coefficient of x^2 in $P(x)$ if all third-order Newton's forward differences are 1

94. The following data are given for a polynomial $P(x)$ of unknown degree

x	0	1	2	3
$P(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order Newton's forward differences are 1

95. The Newton forward difference formula is used to approximate $f(0.3)$ given the following data

x	0.0	0.2	0.4	0.6
$f(x)$	15	21	30	51

Suppose it is discovered that $f(0.4)$ was understated by 10 and $f(0.6)$ was overstated by 5. By what amount should the approximation to $f(0.3)$ be changed?

96. The Bernstein polynomial of degree n for $f \in C[0,1]$ is given by

$$B_n(x) = \sum_{k=0}^{n} \binom{n}{k} f\left(\frac{k}{n}\right) x^k (1-x)^{n-k}$$

Find $B_3(x)$ for the functions

(a) $f(x) = x$

(b) $f(x) = 1$

Show that

$$B_n(x) = \left(\frac{n-1}{n}x^2 + \frac{1}{n}x\right)$$

for $f(x) = x^2$.

97. Show that $H_{2n+1}(x)$ is the unique polynomial of at least degree agreeing with f and f' at $x_0, x_1, \cdots, x_n$.

98. Show that, if

$$z_0 = z_1 = x_0, z_2 = z_3 = x_1$$

and

$$f[z_0, z_1] = f'(x_0), f[z_2, z_3] = f'(x_1)$$

then

$$H_3(x) = f[z_0] + f[z_0, z_1](x - x_0) + f[z_0, z_1, z_2](x - x_0)^2 + f[z_0, z_1, z_2, z_3](x - x_0)^2(x - x_1)$$

99. A clamped cubic spline for a function f is given by

$$S(x) = \begin{cases} 3(x-1) + 2(x-1)^2 - d(x-1)^3 & x \in [1,2] \\ a + b(x-2) + c(x-2)^2 + d(x-2)^3 & x \in [2,3] \end{cases}$$

Given $f'(1) = f'(3)$, then find a, b, c and d.

100. A clamped cubic spline for a function f is given by

$$S(x) = \begin{cases} 1 + ax + 2x^2 - 2x^3 & x \in [1,2] \\ 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3 & x \in [1,2] \end{cases}$$

Find $f'(0)$ and $f'(2)$.