INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
Department of Mathematics and Statistics
MA633K - Numerical Analysis
Numerical Linear Algebra

Note: Usual Notations are used. Questions may have some typos or grammatical mistakes as proofreading is not done for this.

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1. Find the Cholesky Decomposition and then solve the following system

$$\begin{pmatrix} 9.0000 & 0.6000 & -0.3000 & 1.5000 \\ 0.6000 & 16.0400 & 1.1800 & -1.5000 \\ -0.3000 & 1.1800 & 4.1000 & -0.5700 \\ 1.5000 & -1.5000 & -0.5700 & 25.4500 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \\ 1 \end{pmatrix}$$

2. Find the Cholesky decomposition for the following matrix A. Verify that the product of the decomposed matrix and A is same.

$$\begin{pmatrix} 1.4982 & 1.0362 & 1.8581 & 1.3042 \\ 1.0362 & 1.4433 & 1.4397 & 1.2147 \\ 1.8581 & 1.4397 & 2.3994 & 1.8672 \\ 1.3042 & 1.2147 & 1.8672 & 1.9478 \end{pmatrix}$$

3. Using Gauss-Seidel and SOR method ($\omega = 1.4$) solve the following linear system

$$\begin{pmatrix} 7 & 3 & -1 & 2 \\ 3 & 8 & 1 & -4 \\ -1 & 1 & 4 & -1 \\ 2 & -4 & -1 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix}$$

4. Find the range of solutions for the system

$$\begin{aligned} x_1 - x_2 &= 1.4 \pm 0.02 \\ x_1 + x_2 &= 3.8 \pm 0.03, \end{aligned}$$

using Gauss-Jordan method assuming maximum errors in the constants as shown.

5. Solve the following system of equations using Gauss Elimination method:

$$\begin{aligned} x_1 - x_2 + 2x_3 - x_4 &= -8; \\ 2x_1 - 2x_2 + 3x_3 - 3x_4 &= -20; \\ x_1 + x_2 + x_3 &= -2; \\ x_1 - x_2 + 4x_3 + 3x_4 &= 4 \end{aligned}$$

6. Gauss- Seidel method is used to solve the system

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 2; \\ 4x_1 + 9x_2 - 3x_3 &= 8; \\ -2x_1 - 3x_2 + 7x_3 &= 10 \end{aligned}$$

starting with $x^{(0)} = (-0.75, 1.75, 1.75)^\top$. What is the solution obtained at the end of the second iteration?

7. Suppose that the vector b is perturbed to obtain a vector $\tilde{b}$; x and $\tilde{x}$ satisfy $Ax = b$ and $A\tilde{x} = \tilde{b}$. What is M if the relative error in x and the relative error in b are related by $\frac{||x-\tilde{x}||}{||x||} < M\frac{||b-\tilde{b}||}{||b||}$?

8. Using power method, compute the dominant eigenvalue (in magnitude) and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}.$$

Start with an initial vector $\bar{v}_0 = (1, 1, 1)^\top$. Perform 3 steps of computations.

9. Compute the matrix Q and R using the QR Decomposition algorithm for the given matrix A. Verify that $A - QR = 0$

$$\begin{pmatrix} 0.8147 & 0.6324 & 0.9575 & 0.9572 \\ 0.9058 & 0.0975 & 0.9649 & 0.4854 \\ 0.1270 & 0.2785 & 0.1576 & 0.8003 \\ 0.9134 & 0.5469 & 0.9706 & 0.1419 \end{pmatrix}$$

10. Does all 2×2 matrix have LU decomposition? If so, prove. If not, give a counterexample.

11. Find the singular value decomposition and QR decomposition of the following matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

12. As per the iterative algorithm to solve the system $Ax = b$, the SOR method can be written in the form

$$x^{(k)} = Gx^{(k-1)} + c$$

Write the G matrix for SOR method. If

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 4 & 8 & 2 \\ 6 & 2 & 10 \end{bmatrix},$$

then compute the product of the eigenvalues of G for $\omega = 1$

13. Obtain the QR decomposition for the following matrix.

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 0 & 1 & -2 \end{bmatrix}$$

14. Find the LU decomposition of

$$\begin{bmatrix} 3 & 9 \\ 2 & 7 \end{bmatrix}$$

15. Given the LU decomposition

$$\begin{bmatrix} 2 & 3 \\ 8 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$$

use it to solve the linear system

$$\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

16. Find the Doolittle decomposition

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{bmatrix}$$

and the solve for $Ax = (1, 0, 3)^T$

17. Find the LU decomposition

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

and the solve for $Ax = (4, 4, 6)^T$

18. Show that the following matrix cannot be written in the LU form

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{bmatrix}$$

19. Show that the following matrix is invertible but has no LU decomposition

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

20. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Find the center and radius of Gerschgorin's disks associated to A

21. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 7 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

Find the center and radius of Gerschgorin's disks associated to A

22. Use naive Gauss elimination methd to find the solution of the following linear systems

$$A = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 0.6667 & 0.333 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.729 & 0.81 & 0.9 \\ 1 & 1 & 1 \\ 1.331 & 1.21 & 1.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.6867 \\ 0.8338 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & -3 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

23. Consider the following linear systems.

$$\begin{bmatrix} 0.004 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve it

(a) Exactly

(b) by naive Gaussian elimination method using a two digit rounding

(c) interchanging the rows and thenby naive Gaussian elimination method using a two digit rounding

24. Consider the following linear systems.

$$\begin{bmatrix} 1 & 592 \\ 592 & 4308 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 437 \\ 2251 \end{bmatrix}$$

Solve it

(a) Exactly

(b) by naive Gaussian elimination method using a four digit rounding

(c) interchanging the rows and thenby naive Gaussian elimination method using a four digit rounding

25. Consider the following linear systems.

$$\begin{bmatrix} 0.5 & -1 \\ 1.02 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9.5 \\ -18.8 \end{bmatrix}$$

Solve it

(a) Exactly

(b) by naive Gaussian elimination method using a five digit rounding

(c) interchanging the rows and then by naive Gaussian elimination method using a five digit rounding

(d) replace 0.5 by 0.52, repeat above computations and see the difference

26. Calculate the 1-norm, ∞-norm, spectral norm and Frobenius norm of the following matrices

$$\begin{bmatrix} 1 & -7 \\ -2 & -3 \end{bmatrix}, \begin{bmatrix} 2 & -8 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 1 & -3 \end{bmatrix}$$

27. Calculate the condition number of the following matrices (with 1-norm, ∞-norm, spectral norm and Frobenius norm)

$$\begin{bmatrix} 1 & -7 \\ -2 & -3 \end{bmatrix}, \begin{bmatrix} 2 & -8 \\ 3 & 1 \end{bmatrix}$$

28. Calculate the 1-norm, ∞-norm and Frobenius norm of the following matrices

$$\begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 6 & -1 \\ 3 & 1 & 0 \\ 2 & 4 & -7 \end{bmatrix}, \begin{bmatrix} 1 & 7 & 3 \\ 4 & -2 & -2 \\ -2 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 3 \\ 1 & 5 & 6 \\ 2 & -1 & 3 \end{bmatrix}, \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix}$$

29. Calculate the condition number of the following matrices (with the norm specified)

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}, (1 - norm, \text{Frobenius norm}) \begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} (\infty - norm)$$

$$\begin{bmatrix} 4 & -2 \\ 6 & 0 \end{bmatrix}, (1 - norm, \text{Frobenius norm}) \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} (\infty - norm)$$

30. Calculate the condition number of the following matrix using 1-norm

$$\begin{bmatrix} 2 & 4 & -1 \\ 2 & 5 & 2 \\ -1 & -1 & 1 \end{bmatrix} (\infty - norm)$$

31. Calculate the condition number of the following matrix using 1-norm

$$\begin{bmatrix} 1 & 10^4 \\ 2 & 3 \end{bmatrix} (\infty - norm)$$

Conclude that the problem as stated is ill-conditioned

32. Calculate the condition number of the following matrix using ∞-norm

$$\begin{bmatrix} 1.01 & 0.99 \\ 0.099 & 1.01 \end{bmatrix} (\infty - norm)$$

33. Calculate the condition number of the following matrix using spectral norm

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} (\infty - norm)$$

34. Calculate the condition number of the following system using spectral norm

$$\begin{aligned} 3x_1 + 4x_2 &= 7 \\ 5x_1 - 2x_2 &= 3 \end{aligned}$$

Solve the system and show that the solution is $(1, 1)$. Change the right hand side as $(7.005, 2.991)$ and find the solution. Can you conclude that this system is well-conditioned?

35. Calculate the condition number of the following system using spectral norm

$$\begin{aligned} x_1 + x_2 &= 2 \\ 1.01x_1 + x_2 &= 2.01 \end{aligned}$$

Solve the system and show that the solution is $(1, 1)$. Change the right hand side as $(2.005, 2.005)$ and find the solution. Can you conclude that this system is ill-conditioned?

36. Calculate the condition number of the following system using spectral norm

$$\begin{aligned} x_1 + x_2 &= 2 \\ 1.01x_1 + x_2 &= 2.01 \end{aligned}$$

Solve the system and show that the solution is $(1, 1)$. Change the system as

$$\begin{aligned} x_1 + x_2 &= 2 \\ 1.0001x_1 + x_2 &= 2.01 \end{aligned}$$

. Solve this new system. Can you conclude that this system is ill-conditioned?

37. If $k(A) = 10^6$ and you solve $Ax = b$ on a computer with 7 significant digits (base 10), What is the expected number of significant digits of accuracy of the solution?

38. Prove that condition number of an orthogonal matrix with respect to the spectral norm is 1.

39. Use Gauss elimination to decompose the following system as Doolittle decomposition and then solve the system

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27; \\ -3x_1 + 6x_2 - 2x_3 &= -61.5; \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

40. Use Doolittle decomposition to determine the inverse of the matrix for the following system

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27; \\ -3x_1 + 6x_2 - 2x_3 &= -61.5; \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

41. Compute Crout decomposition for the following system and the solve the system

$$\begin{aligned} 2x_1 - 5x_2 + x_3 &= 12; \\ -x_1 + 3x_2 - x_3 &= -8; \\ 3x_1 - 4x_2 + 2x_3 &= 16 \end{aligned}$$

42. Use Gauss elimination to decompose the following system as Doolittle decomposition and then solve the system

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 12; \\ -3x_1 + 6x_2 - 2x_3 &= 18; \\ x_1 + x_2 + 5x_3 &= -6 \end{aligned}$$

43. Solve the following system of equation by LU decomposition

$$\begin{aligned} 8x_1 + 4x_2 - x_3 &= 11; \\ -2x_1 + 5x_2 + x_3 &= 4; \\ 2x_1 - x_2 + 6x_3 &= 7 \end{aligned}$$

Find the matrix inverse as explained in the class

44. Solve the following system of equation by LU decomposition

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38; \\ -3x_1 - x_2 + 7x_3 &= -34; \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

45. Solve the following system of equation by Gaussian elimination method without pivoting and then with partial pivoting.

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38; \\ -3x_1 - x_2 + 7x_3 &= -34; \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

46. The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day),

$$\begin{aligned} 15c_1 - 3c_2 - c_3 &= 3800; \\ -3c_1 + 18c_2 - 6c_3 &= 1200; \\ -4c_1 - c_2 + 12c_3 &= 2350 \end{aligned}$$

(a) Use Doolittle decomposition to find the solution

(b) Use Crout decomposition to find the solution

(c) Use Gaussian elimination to find the solution

(d) Determine how much the rate of mass input to reactor 3 must be increased to induce a 10 g/m^3 rise in the concentration of reactor 1.

(e) How much will the concentration in reactor 3 be reduced if the rate of mass input to reactors 1 and 2 is reduced by 500 and 250 g/day, respectively?

47. Determine the LU decomposition of the following matrix and then obtain the determinant of the matrix

$$\begin{bmatrix} 8 & 2 & 1 \\ 3 & 7 & 2 \\ 2 & 3 & 9 \end{bmatrix}$$

48. Determine the condition number of the following matrix using the column-sum norm

$$\begin{bmatrix} 0.125 & 0.25 & 0.5 & 1 \\ 0.15625 & 0.625 & 0.25 & 1 \\ 0.00463 & 0.02777 & 0.16667 & 1 \\ 0.001953 & 0.015625 & 0.125 & 1 \end{bmatrix}$$

49. Determine the condition number of the following matrix using the row-sum norm

$$\begin{bmatrix} 1 & 4 & 9 & 16 & 25 \\ 4 & 9 & 16 & 25 & 36 \\ 9 & 16 & 25 & 36 & 49 \\ 16 & 25 & 36 & 49 & 64 \\ 25 & 36 & 49 & 64 & 81 \end{bmatrix}$$

50. The following Vandermonde matrix is also ill-conditioned.

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}$$

Determine the condition number based on the row-sum norm for the case where $x_1 = 4, x_2 = 2$, and $x_3 = 7$.

51. Consider vector

$$\begin{aligned} \vec{A} &= 2\vec{i} - 3\vec{j} + a\vec{k} \\ \vec{B} &= b\vec{i} + \vec{j} - 4\vec{k} \\ \vec{C} &= 3\vec{i} + c\vec{j} + 2\vec{k} \end{aligned}$$

Vector $\vec{A}$ is perpendicular to $\vec{B}$ as well as to $\vec{C}$. If $\vec{B}.\vec{C} = 2$, use Doolittle decomposition to solve for three unknowns a, b and c.

52. Consider vector

$$\begin{aligned} \vec{A} &= a\vec{i} + b\vec{j} + c\vec{k} \\ \vec{B} &= -2\vec{i} + \vec{j} - 4\vec{k} \\ \vec{C} &= \vec{i} + 3\vec{j} + 2\vec{k} \end{aligned}$$

Where $\vec{A}$ is an unknown vector. If

$$(\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C}) = (5a + 6)\vec{i} + (3b - 2)\vec{j} + (-4c + 1)\vec{k}$$

, use Doolittle decomposition to solve for three unknowns a, b and c.

53. Find the Cholesky decomposition for the following matrix

$$\begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix}$$

54. Find the Cholesky decomposition for the following matrix

$$\begin{bmatrix} 8 & 20 & 15 \\ 20 & 80 & 50 \\ 15 & 50 & 60 \end{bmatrix}$$

55. Use the Gauss-Seidel method to obtain the solution of the system

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

56. Use the Richardson method to obtain the solution of the system

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

57. Use the Jacobi method to obtain the solution of the system

$$\begin{aligned} 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

58. The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day),

$$\begin{aligned} 15c_1 - 3c_2 - c_3 &= 3800; \\ -3c_1 + 18c_2 - 6c_3 &= 1200; \\ -4c_1 - c_2 + 12c_3 &= 2350 \end{aligned}$$

Solve this problem using Gauss-Seidel method to $\epsilon_s = 5\%$

59. Use Gauss-Seidel method to solve the system until the relative error $\epsilon_s = 5\%$

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27; \\ -3x_1 + 6x_2 - 2x_3 &= -61.5; \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

60. Use SOR method with $\omega = 0.95$ to solve the system until the relative error $\epsilon_s = 5\%$

$$\begin{aligned} -3x_1 + x_2 + 12x_3 &= 50 \\ 6x_1 - x_2 - x_3 &= 3 \\ 6x_1 + 9x_2 + x_3 &= 40 \end{aligned}$$

61. Use SOR method with $\omega = 1.2$ to solve the system until the relative error $\epsilon_s = 5\%$

$$\begin{aligned} 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \\ -8x_1 + x_2 - 2x_3 &= -20 \end{aligned}$$

62. Employ the power method to determine the highest eigenvalue for the following matrix (first 5 iterations)

$$\begin{bmatrix} 3.556 & -1.778 & 0 \\ -1.778 & 3.556 & -1.778 \\ 0 & -1.778 & 3.556 \end{bmatrix}$$

63. Divide the following matrix by 0.5625 and then find the inverse. Employ the power method to determine the highest eigenvalue for the following matrix (first 3 iterations)

$$\begin{bmatrix} 3.556 & -1.778 & 0 \\ -1.778 & 3.556 & -1.778 \\ 0 & -1.778 & 3.556 \end{bmatrix}$$

64. Employ the power method to determine the highest eigenvalue for the following matrix (first 5 iterations)

$$\begin{bmatrix} 2 & 8 & 10 \\ 8 & 4 & 5 \\ 10 & 5 & 7 \end{bmatrix}$$

65. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} -1 & 3 \\ 1 & 5 \end{bmatrix}$$

66. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

67. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

68. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 0 & 1 & -2 \end{bmatrix}$$

69. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

70. Compute the QR decomposition of the following matrix

$$\begin{bmatrix} -1 & -1 & 1 \\ 1 & 3 & 3 \\ -1 & -1 & 5 \\ 1 & 3 & 7 \end{bmatrix}$$

71. Consider the matrix

$$\begin{bmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{bmatrix}$$

Compute the eigenvalue of this matrix using power method with $x^{(0)} = (1, 0.5, 0.25)^T$ and 10 iterations. And then compute the eigenvalue of this matrix with $x^{(0)} = (0, 0.5, 0.25)$ and 3 iterations

72. Use Gaussian elimination method (both with and without partial pivoting) to find the solution of the linear system

$$\begin{aligned} 6x_1 + 2x_2 + 2x_3 &= -2 \\ 2x_1 + 0.6667x_2 + 0.3333x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

73. Use Gaussian elimination method (both with and without partial pivoting) to find the solution of the linear system

$$\begin{aligned} 0.729x_1 + 0.81x_2 + 0.9x_3 &= 0.6867 \\ x_1 + x_2 + x_3 &= 0.8338 \\ 1.331x_1 + 1.21x_2 - 1.1x_3 &= 1 \end{aligned}$$

74. Use Gaussian elimination method (both with and without partial pivoting) to find the solution of the linear system

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 2 \\ 3x_1 - 3x_2 + x_3 &= -1 \\ x_1 + x_2 &= 3 \end{aligned}$$

75. Solve the system

$$\begin{aligned} 0.004x_1 + x_2 &= 1 \\ x_1 + x_2 &= 2 \end{aligned}$$

(a) exactly,

(b) by Gaussian elimination using a two digit rounding calculator, and

(c) interchanging the equations and then solving by Gaussian elimination using a two digit rounding calculator

76. Solve the following system by Gaussian elimination, first without row interchanges and then with row interchanges, using four-digit rounding arithmetic

$$\begin{aligned} x_1 + 592x_2 &= 437 \\ 592x_1 + 4308x_2 &= 2251 \end{aligned}$$

77. Solve the system

$$\begin{aligned} 0.5x_1 - x_2 &= -9.5 \\ 1.02x_1 - 2x_2 &= -18.8 \end{aligned}$$

(a) Using Gaussian elimination method

(b) Solve the system by modifying 0.5 to 0.52

In both the cases, use rounding upto 5 digits after decimal point. Obtain the residual error in each case

78. The matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

has eigenvalues 2, 1 and 1 and the corresponding eigenvectors $(1,2,3)', (0,1,2)'$ and $(0,2,1)'$. Perform 3 iterations to find the eigen value and the corresponding eigenvector to which the power method converge when we start with the iteration $x^{(0)} = (0, 0.5, 0.75)'$. Repeat it for $x^{(0)} = (0.001, 0.5, 0.75)'$

79. The matrix

$$\begin{bmatrix} 5.4 & 0 & 0 \\ 113.0233 & -0.5388 & -0.6461 \\ -46.0567 & -6.4358 & -0.9612 \end{bmatrix}$$

has eigenvalues 5.4, 1.3 and -2.8 and the corresponding eigenvectors $(0.2, -4.1, 2.7)', (0, 1.3, -3.7)'$ and $(0, 2.6, 9.1)'$. Perform 3 iterations to find the eigen value and the corresponding eigenvector to which the power method converge when we start with the iteration $x^{(0)} = (0, 1, 1)$

80. The matrix

$$\begin{bmatrix} 0.7825 & 0.8154 & -0.1897 \\ -0.3676 & 2.2462 & -0.0573 \\ -0.1838 & 0.1231 & 1.9714 \end{bmatrix}$$

has eigenvalues 2, 2 and 1. Perform 5 iterations to find the eigen value and the corresponding eigenvector to which the power method converge when we start with the iteration $x^{(0)} = (1, 3, 6)'$

81. Study the convergence of the Jacobi and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} 5x_1 + 2x_2 + x_3 &= 0.12 \\ 1.75x_1 + 7x_2 + 0.5x_3 &= 0.1 \\ x_1 + 0.2x_2 + 4.5x_3 &= 0.5 \end{aligned}$$

82. Study the convergence of the Jacobi and the Richardson method for the following systems by starting with $x_0 = (0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 1 \\ -x_1 + x_2 - x_3 &= 1 \\ -2x_1 - 2x_2 + x_3 &= 1 \end{aligned}$$

83. Find the first two iterations of the Jacobi and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0)'$

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1 \\ 3x_1 + 6x_2 + 2x_3 &= 0 \\ 3x_1 + 3x_2 + 7x_3 &= 4 \end{aligned}$$

84. Find the first two iterations of the Jacobi and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0, 0)'$

$$\begin{aligned} 10x_1 + 5x_2 &= 6 \\ 5x_1 + 10x_2 - 4x_3 &= 25 \\ -4x_2 + 8x_3 - x_4 &= -11 \\ -x_3 + 5x_4 &= -11 \end{aligned}$$

85. Find the first two iterations of the Jacobi and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0, 0, 0)'$

$$\begin{aligned} 4x_1 + x_2 + x_3 + x_5 &= 6 \\ -x_1 - 3x_2 + x_3 + x_4 &= 6 \\ 2x_1 + x_2 + 5x_3 - x_4 - x_5 &= 6 \\ -x_1 - x_2 - x_3 + 4x_4 &= 6 \\ 2x_2 - x_3 + x_4 + 4x_5 &= 6 \end{aligned}$$

86. Study the convergence of the Richardson and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} x_1 + x_2 + 10x_3 &= -1 \\ 2x_1 + 3x_2 + 5x_3 &= -6 \\ 3x_1 + 2x_2 - 3x_3 &= 4 \end{aligned}$$

87. Study the convergence of the Richardson and the Jacobi method for the following systems by starting with $x_0 = (0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} -x_1 + 5x_2 - 2x_3 &= 3 \\ x_1 + x_2 - 4x_3 &= -9 \\ 4x_1 - x_2 + 2x_3 &= 8 \end{aligned}$$

88. Study the convergence of the Jacobi and the Gauss-Seidel method for the following systems by starting with $x_0 = (0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} x_1 + 0.5x_2 + 0.5x_3 &= 1 \\ 0.5x_1 + x_2 + 0.5x_3 &= 8 \\ 0.5x_1 + 0.5x_2 + x_3 &= 1 \end{aligned}$$

89. Study the convergence of the Jacobi method for the following systems by starting with $x_0 = (0, 0, 0, 0)'$ and performing three iterations:

$$\begin{aligned} 10x_1 - x_2 + 2x_3 - 3x_4 &= 0 \\ x_1 + 10x_2 - x_3 + 2x_4 &= 5 \\ 2x_1 + 3x_2 + 20x_3 - x_4 &= -10 \\ 3x_1 + 2x_2 + x_3 + 20x_4 &= 15 \end{aligned}$$

90. Find the number of operations $(+, /, -, *)$ for LU factorization algorithm

91. Find the number of operations $(+, /, -, *)$ for solving $Ly = b$ requires

92. Determine the LDL' and Cholesky factorization of the positive definite matrices

$$\begin{bmatrix} 4 & -1 & 1 \\ -1 & 4.25 & 2.75 \\ 1 & 2.75 & 3.5 \end{bmatrix}, \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}, \begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 5 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & 0 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & 4 \end{bmatrix}$$

93. Find the number of operations required for the Cholesky factorization algorithm required.

94. Find the optimal choice of ω for the SOR method for the matrix

$$\begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

95. Find the first two iterations of the SOR method with $\omega = 1.1$ and then for $\omega = 1.2, \omega = 1.3$ for the following linear systems by starting with $x_0 = (0, 0, 0)'$

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5 \\ -x_1 + 3x_2 + x_3 &= -4 \\ 2x_1 + 2x_2 + 5x_3 &= 1 \end{aligned}$$

96. Find the first two iterations of the SOR method with $\omega = 1.1$ and then for $\omega = 1.2, \omega = 1.3$ for the following linear systems by starting with $x_0 = (0,0,0,0)'$

$$\begin{aligned} 4x_1 + x_2 - x_3 + x_4 &= -2 \\ x_1 + 4x_2 - x_3 - x_4 &= -1 \\ -x_1 - x_2 + 5x_3 + x_4 &= 0 \\ x_1 - x_2 + x_3 + 3x_4 &= 1 \end{aligned}$$

97. Find a singular value decomposition for the following matrices

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

98. Find a singular value decomposition for the following matrices

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

99. Find a singular value decomposition for the following matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix}$$

100. Let $A^{-1} = A = A^T$. Given any orthogonal matrix U, find an orthogonal matrix V such that $A = U\Sigma V^T$ is an SVD for A. If

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, U_1 = \frac{1}{5}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}, U_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

find V_1 and V_2 such that $A = U_1\Sigma_1 V_1 = U_2\Sigma_2 V_2$