INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
Department of Mathematics and Statistics
MA633L - Numerical Analysis
100 Problems on Numerical Differentiation

Note: Usual Notations are used. Questions may have some typos or grammatical mistakes as proofreading is not done for this.

---

1. Using Backward/Forward Euler method, find the y_5 for the following IVPs.

   (a) $y' = t^2 + y^3, y(0) = 0, 0 \leq t \leq 1$

   (b) $y' = e^y + y, y(0) = 0, 0 \leq t \leq 1$

   (c) $y' = y^2 - \cos(y), y(0) = 0, 0 \leq t \leq 1$

2. Using Backward/Forward Euler method, find the y_5 for the following IVPs. Here $h = 1/100$.

   (a) $y' = y + y^2, y(1) = e/(16 - e), t \in [1, 2.77]$

   (b) $y' = t + y^2, y(0) = 1, t \in [0, 0.9]$

   (c) $y' = y - t, y(1) = 1, t \in [1, 1.75]$

   (d) $y' = yt + y^2t^2, y(2) = -0.63966, t \in [2, 5]$

3. Using Taylor series method of order $n = 2$ and $n = 3$, find the y_5 for the following IVPs. Here $h = 1/100$.

   (a) $y' = y + y^2, y(1) = e/(16 - e), t \in [1, 2.77]$

   (b) $y' = t + y^2, y(0) = 1, t \in [0, 0.9]$

   (c) $y' = y - t, y(1) = 1, t \in [1, 1.75]$

   (d) $y' = yt + y^2t^2, y(2) = -0.63966, t \in [2, 5]$

4. Using Backward/Forward Euler method, find the approximate solutions for each of the following IVPs

   (a) $y' = te^{3t} - 2y, t \in [0, 1], y(0) = 0, h = 0.5$

   (b) $y' = 1 + (t - y)^2, t \in [2, 3], y(2) = 1, h = 0.5$

   (c) $y' = 1 + y/t, t \in [1, 2], y(1) = 2, h = 0.25$

   (d) $y' = \cos(2t) + \sin(3t), t \in [0, 1], y(0) = 1, h = 0.25$

5. Using Taylor series of order two, find the approximate solutions for each of the following IVPs

   (a) $y' = e^{t-y}, t \in [0, 1], y(0) = 0, h = 0.5$

   (b) $y' = \frac{1+t}{1+y}, t \in [1, 2], y(1) = 2, h = 0.5$

(c) $y' = -y + ty^{1/2}, t \in [2,3], y(2) = 2, h = 0.25$

(d) $y' = t^{-2}(\sin(2 - t - 2ty), t \in [1,2], y(1) = 2, h = 0.25$

6. Using RK2/RK3/RK4 methods, find the approximate solutions for each of the following IVPs

   (a) $y' = y/t - (y/t)^2, t \in [0,1], y(0) = 0, h = 0.5$

   (b) $y' = 1 + (t - y)^2, t \in [2,3], y(2) = 1, h = 0.5$

   (c) $y' = 1 + y/t, t \in [1,2], y(1) = 2, h = 0.25$

   (d) $y' = \cos(2t) + \sin(3t), t \in [0,1], y(0) = 1, h = 0.25$

7. Using Predictor-Corrector methods, find the approximate solutions for each of the following IVPs

   (a) $y' = te^{3t} - 2y, t \in [0,1], y(0) = 0, h = 0.5$

   (b) $y' = 1 + (t - y)^2, t \in [2,3], y(2) = 1, h = 0.5$

   (c) $y' = 1 + y/t, t \in [1,2], y(1) = 2, h = 0.25$

   (d) $y' = \cos(2t) + \sin(3t), t \in [0,1], y(0) = 1, h = 0.25$

8. Using linear shooting method, find the approximate solutions for each of the following BVPs

   (a) $y'' = -3y' + 2y + 2x + 3, 0 \le x \le 1, y(0) = 2, y(1) = 1, h = 0.1$

   (b) $y'' = -4/xy' + 2/x^2y - 2/x^2 \ln x, 1 \le x \le 2, y(1) = -1/2, y(2) = \ln 2, h = 0.05$

   (c) $y'' = -(x+1)y' + 2y + (1 - x^2)e^{-x}, 1 \le x \le 2, y(1) = -1/2, y(2) = \ln 2, h = 0.05$

   (d) $y'' = 1/xy' + 3/x^2y + 1/x \ln x - 1, 1 \le x \le 2, y(1) = y(2) = 0, h = 0.05$

9. Using nonlinear shooting method, find the approximate solutions for each of the following BVPs

   (a) $y'' = 2y^3 - 6y - 2x^3, 1 \le x \le 2, y(1) = 2, y(2) = 5/2, h = 0.2$

   (b) $y'' = y' - 2(y - \ln x)^2 - 1/x, 2 \le x \le 3, y(2) = 1/2 + \ln 2, y(3) = 1/3 + \ln 3, h = 0.2$

   (c) $y'' = \frac{x^2y'^2 - 9y^2 + 4x^6}{x^5}, 1 \le x \le 2, y(1) = 0, y(2) = \ln 256, h = 0.1$

10. Using finite difference method, find the approximate solutions for each of the following BVPs

    (a) $y'' = 4(y - x), 0 \le x \le 1, y(0) = 0, y(1) = 2, h = 0.2$

    (b) $y'' = y' + 2y + \cos x, 0 \le x \le \frac{\pi}{2}, y(0) = -0.3, y(\pi/2) = -0.1, h = 0.2$

    (c) $y'' = -3y' + 2y + 2x + 3, 0 \le x \le 1, y(0) = 2, y(1) = 1, h = 0.1$