MA633L-Numerical Analysis

Lecture 1 : Introduction - What? Why? How?

Panchatcharam Mariappan[1]

[1]Associate Professor
Department of Mathematics and Statistics
IIT Tirupati, Tirupati

January 9, 2025

भारतीय प्रौद्योगिकी संस्थान तिरुपति
IIT
TIRUPATI

About the Course

Introduction

- Solving mathematical problems require more practical effort
- Listen to Lectures, Read Books and Try to Solve Problems
- If you do not solve problems, you delude yourself that you understand everything, you will be awakened only when first problem you attempt in Quiz-1 or Test-1
- Enough problems are given at the end of each class. Work out those problems before the next class

Introduction

- Work with this course material, think about it, write your own comments, solve problems and make yourself comfortable. If you follow this, problems will be familiar to you like finding your home in a street where you live as you wandered around your home many times, otherwise, solving the problem will be like trying to locate your home from the map locations.

Introduction

- Instructor helps you to understand each topic thoroughly, rather than remember how to go through the mechanics of a proof or calculation
- If you memorize the proof and calculation, it will be retained in your brain until examination and will be erased afterwards

Introduction

- Attend lectures, reread the lecture notes, correct mistakes of them
- Read reference books and text books, correlate with lecture notes
- Solve problems from lecture notes, books, past exam papers
- Discuss the subject with your friends and colleagues
- Convincingly explain the topic to your friends so that you will really understand it yourself.

Marks (Tentative)

Component	Marks	Topics
Test-1	20	Till Jan 31, 2025 Covered
Test-2	20	Till Feb 22, 2025 Covered
End Semester Theory	60	All : 3 Hours Exam

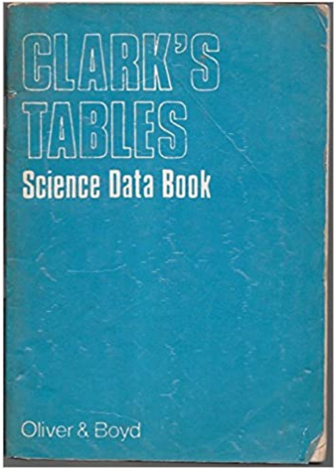
Numerical Analysis: What? Why? How?

Numerical Analysis

- A study of algorithms that use numerical approximation for the problems of mathematical analysis.
- Area of mathematics and computer science that creates, analyzes, and implements algorithms for solving numerically the problems of mathematics.
- Problems originate from real world applications, engineering, natural sciences, medicine and so on.

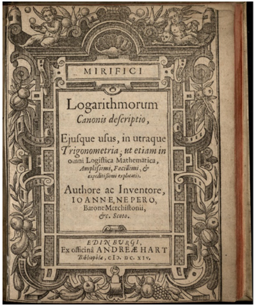
Numerical Analysis: School Memories

- Numerical Approach to Natural Logarithms



Numerical Analysis: School Memories

- Numerical Approach to Natural Logarithms:


MIRIFICI
Logarithmorum
Canonis descriptio,
Ejusque usus, in utraque
Trigonometria; ut etiam in
omni Logistica Mathematica,
Amplissimi, Facillimi, &
expeditissimi explicatio.
Authore ac Inventore,
IOANNE, NEPERO,
Barone Merchistonii,
&c. Scoto.
EDINBURGI,
Ex officinâ ANDREÆ HART
Bibliopôlæ, CIↃ. DC. XIV.


MIRIFICI
Logarithmorum canonis descri-
ptio, eiusque usus in utrâque Trigonome-
tria, ut etiam in omni Logistica mathematica,
amplissimi, facillimi, & expe-
ditissimi explicatio.
LIBER I.

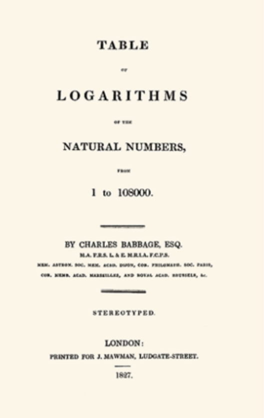
CAPVT. I.
De Definitionibus.

1.Def. Linea æqualiter crescere dicitur, quum punctus eam describens, æqualibus momentis per æqualia intervalla progreditur.

Sit punctus A, à quo ducenda sit linea fluxu alterius puncti, qui sit B. fluat ergo primo momento B ab A in C.
B Secun-

Numerical Analysis: School Memories

- Numerical Approach to Natural Logarithms: Greatest Discovery of 16th Century



TABLE

LOGARITHMS

NATURAL NUMBERS,

1 to 108000.

BY CHARLES BABBAGE, ESQ.

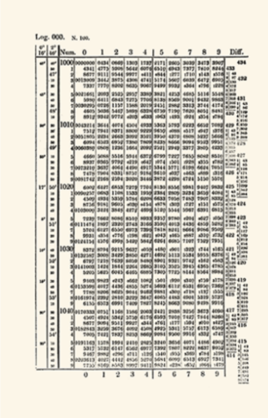
M.A. F.R.S. L. & E. M.R.I.A. F.C.P.S.

STEREOTYPED.

LONDON:

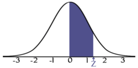
PRINTED FOR J. MAWMAN, LUDGATE-STREET.

1827.



Numerical Analysis: School Memories

- Normal Distribution Table



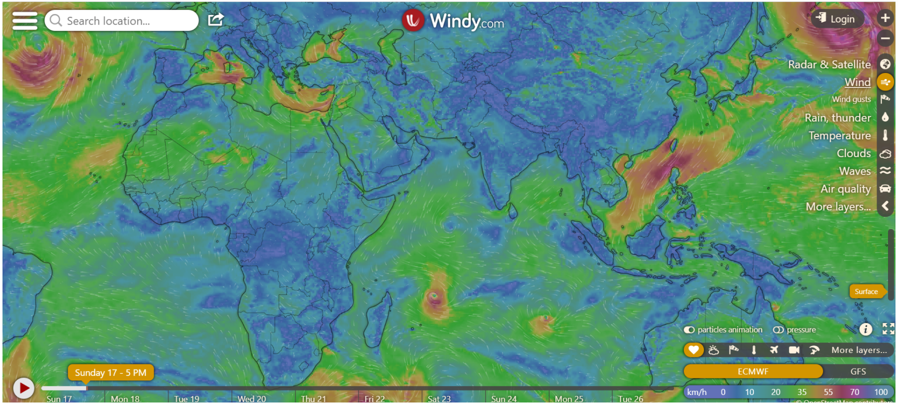
STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Numerical Weather Prediction

- Weather Prediction

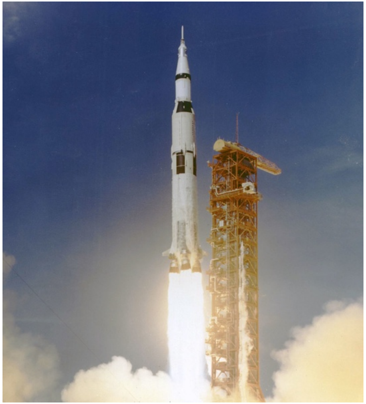


Numerical Analysis: Car Crash

- Car Crash using Finite Element Analysis

Numerical Analysis: Rocket/Missile

- Trajectories of Rocket and Missiles



Numerical Analysis: Applications

- Stock Market
- Aerospace Industry
- All Engineering Disciplines
- Most of the Medical Industries
- Actuarial Analysis
- $\vdots$

Numerical Analysis: Areas

- Numerical Solution of System of Linear Equations
- Numerical Solution of System of Nonlinear Equations
- Numerical Optimization
- Numerical Integration
- Numerical Differentiation
- Numerical Interpolation
- Best approximations
- Wavelets
- $\vdots$

Numerical Analysis: History

Historical Background

- Root finding Method: 1650 BC, Egyptian Rhind Papyrus
- Caculating Length, Areas and Volume of Geometric Figures: 400-350 BC Eudoxus, 285 BC Archimedes
- Greek Mathematicians: Trigonometric functions, Chord of a circle to the arc it subtends, Chord function, Hipparchus (140 BC), Ptolemy (140)

“A logarithmic table is a small table by which we can obtain a knowledge of all geometrical dimensions and motions in space, by a very easy calculation.”

–John Napier

Historical Background

- Solving the Cubic equation $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$, Al-Kashi (1400)
- Properties of Exponents: Stifel (1487-1567)
- Logarithms Table: John Napier (1550-1617), Burgi (1552-1634)
- Calculus Development : Newton (1642-1727), Leibniz (1646-1716)
- Mechanization: Charles Babbage (1791-1871)

Historical Background

- Polynomial Interpolation: Newton (1642-1727)
- Lagrange Interpolation: Lagrange (1736-1813)
- Other mathematicians: Gauss, Euler, Cholesky, Jacobi, Adams-Bashforth and so on

For detailed history of each topic, please refer the book: H.H. Goldstine, A History of Numerical Analysis from the 16th through the 19th Century, Springer, 1977.

Numerical Analysis in India

- Sidharacharya (750 AD) solving a quadratic equation
- Approximate value of π, Aryabhat (5th Century)
- Value of π. Madhava of Sagamagrama (14th Century), Later $\sin x$ and $\cos x$ series.
- Finding Square root of a number, Brahmagupta (628 CE)

Prerequisites

Numerical Analysis: Prerequisites

- Real Analysis
- Riemann Integral
- Linear Algebra
- Differential Equations
- Functional Analysis [For Advanced Topics]

Numerical Analysis: Keywords

- Error: Roundoff, truncation, bounds
- Stability, sensitive to data
- Ill-posed problem
- Conditioning
- Iteration, algorithm
- Convergence

Limit

Definition 1 (Limit)

A function f defined on a set X of a real numbers has the limit L at x_0, written as

$$\lim_{x \to x_0} f(x) = L$$

if, for any given real number $\epsilon > 0$, there exists a real number $\delta > 0$ such that

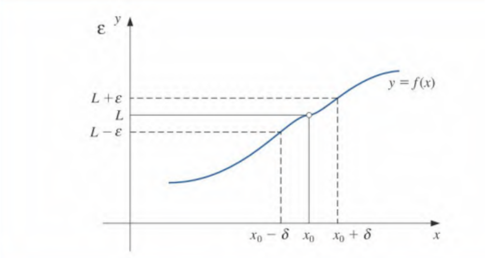
$$|f(x) - L| < \epsilon, \quad \text{whenever} \quad x \in X \quad \text{and} \quad 0 < |x - x_0| < \delta$$

Continuous

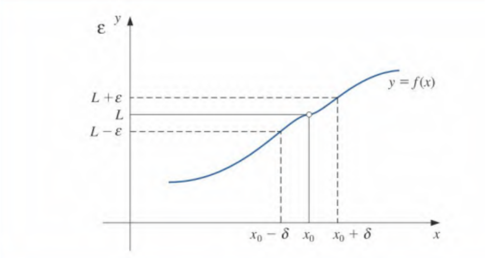
Definition 2 (Continuous)
Let f be a function defined on a set X of real numbers and $x_0 \in X$. Then f is continuous at x_0 if

$$\lim_{x \to x_0} f(x) = f(x_0)$$

The function is continuous on the set X if it is continuous at each number in X.



Set of all Continuous functions



Remarks:

1. The set of all functions that are continuous on a set X is denoted by $C(X)$.
2. The set of all functions that are continuous at every real number is denoted by $C(\mathbb{R})$.

Convergent Sequence

Definition 3

Let $\{x_n\}_{n=1}^{\infty}$ be an infinite sequence of real numbers. This sequence has the limit x or converges to x if, for any $\epsilon > 0$, there exists a positive integer $N(\epsilon)$ such that $|x_n - x| < \epsilon$ whenever $n > N(\epsilon)$.

$$\lim_{n \to \infty} x_n = x$$

means that the sequence converges to x.

Convergent Sequence

Example 4

$$\begin{aligned}\lim_{n\to\infty} \frac{n+1}{n} &= 1\\ \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n &= e\end{aligned}$$

Theorem

Theorem 5

If f is a function defined on a set X of real numbers and $x_0 \in X$, then the following statements are equivalent

1. f is continuous at x_0
2. If $\{x_n\}_{n=1}^{\infty}$ is any sequence in X converging to x_0, then

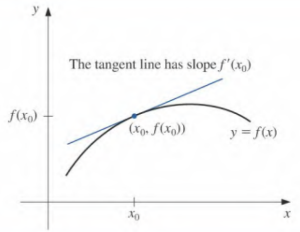
$$\lim_{n \to \infty} f(x_n) = f(x_0)$$

Differentiable

Definition 6 (Derivative)
Let f be a function defined in an open interval containing x_0. The function f is differentiable at x_0 if

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists. The number $f'(x_0)$ is called the derivative of f at x_0. A function that has a derivative at each number in a set X is differentiable on X.



Source: Thomas Calculus Book

Theorem and Remarks

Theorem 7

If the function f is differentiable at x_0, then f is continuous at x_0.

Remark:

1. The set of all functions that have n continuous derivatives on a set X is denoted by $C^n(X)$.
2. The set of all functions that have continuous derivatives of all order at every real number is denoted by $C^\infty(\mathbb{R})$.
3. $C^1(\mathbb{R}) \subset C(\mathbb{R})$. Example, $|x| \in C(\mathbb{R})$, $|x| \notin C^1(\mathbb{R})$.
4. $C^\infty(\mathbb{R}) \cdots \subset C^2(\mathbb{R}) \subset C^1(\mathbb{R}) \subset C(\mathbb{R})$. Example: $e^x \in C^\infty(\mathbb{R})$.

Thanks

Doubts and Suggestions

panch.m@iittp.ac.in

MA633L-Numerical Analysis

Lecture 1 : Introduction - What? Why? How?

Panchatcharam Mariappan[1]

[1]Associate Professor
Department of Mathematics and Statistics
IIT Tirupati, Tirupati

January 9, 2025

भारतीय प्रौद्योगिकी संस्थान तिरुपति
TIRUPATI