MA633L-Numerical Analysis

Lecture 14 : Spline Interpolation

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Runge's Phenomenon

Runge's Phenomenon

Polynomial Interpolation:

- When using high-degree polynomial interpolation over evenly spaced data points, you can experience oscillations

Consider

$$f(x) = \frac{1}{1+25x^2}, x \in [-1, 1]$$

When we interpolate it using polynomial interpolation, you can see large oscillations. This was observed by German mathematician Carl Runge (1901). It is referred as **Runge's phenomenon**

Runge's Phenomenon

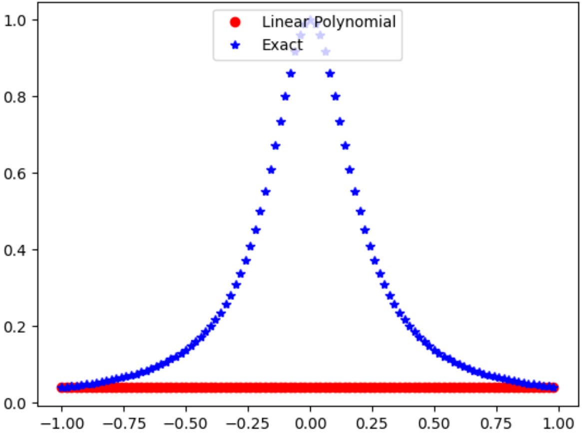


Figure 1: Runge's Phenomenon

Runge's Phenomenon

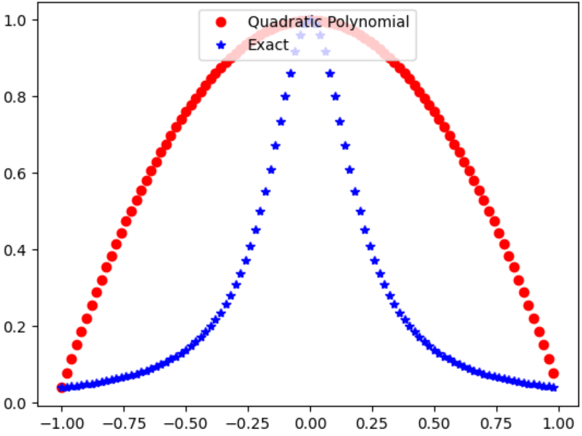


Figure 2: Runge's Phenomenon

Runge's Phenomenon

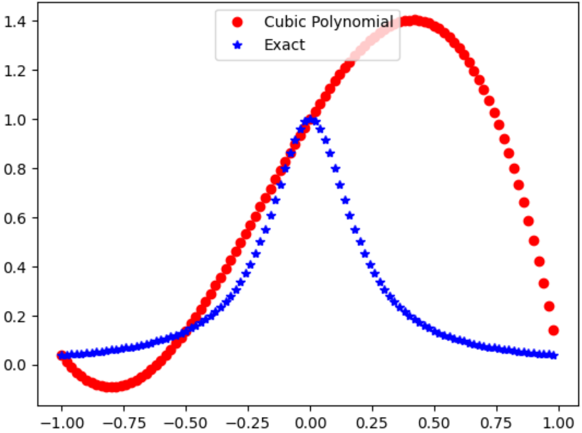


Figure 3: Runge's Phenomenon

Runge’s Phenomenon

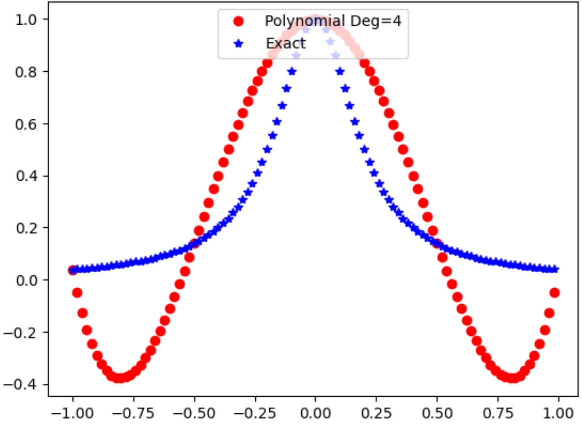


Figure 4: Runge’s Phenomenon

Runge's Phenomenon

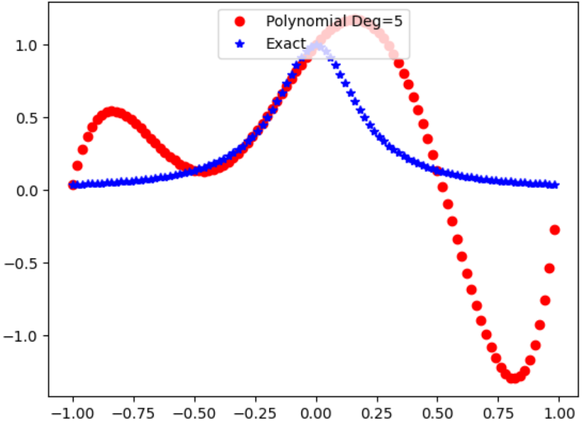


Figure 5: Runge's Phenomenon

Runge’s Phenomenon

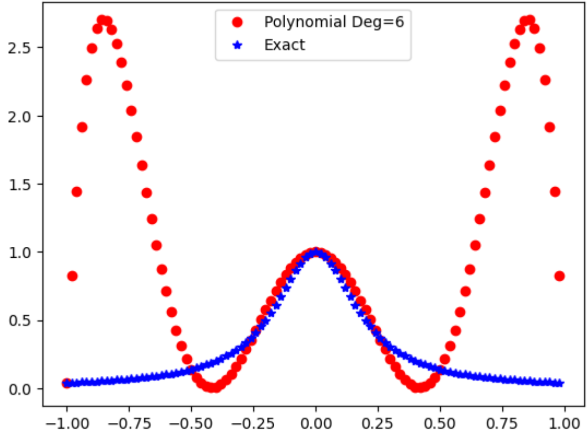


Figure 6: Runge’s Phenomenon

Runge's Phenomenon

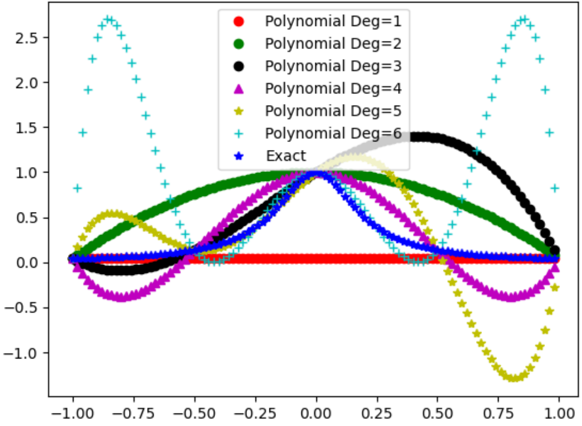


Figure 7: Runge's Phenomenon

Spline Interpolation

Spline Interpolation

Polynomial Interpolation:

- nth-order polynomials were used to interpolate between $n+1$ data points.
- This polynomial captures all $n+1$ points.
- However, it can lead to erroneous results due to round-off error and overshoot.
- Also, high degree polynomials suffer from Runge's phenomenon (oscillations at endpoints)

Spline Interpolation:

- Alternatively, we can apply low-order polynomials to subset of data points.
- Such connecting polynomials are called **spline functions**.

Spline Interpolation: History

- **Fairing** Rooted in the work of draftsman. Needed to draw a gently turning curve between points on drawing.
- French Curve, adhoc devices, made of plastic and presenting a number of curves of different curvature for the draftsman to select.

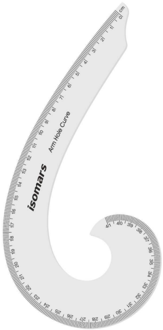


Fig. 6. Kurvenlineale von Gebrüder Wichmann, Berlin.

Spline Interpolation: History

- Long strips of wood were used. It was made to pass through control points by weights laid on the draftsman’s table and attached to the strips. The weights are called ducks and strips of wood are called splines (1891)
- The elastic nature of the strips allowed them to bend only a little while still passing through the prescribed points.
- The first book giving a systematic exposition of spline theory by Ahlberg, Nilson and Walsh [1967]

Spline Interpolation: Applications

- Computer Graphics: Smooth Curve Modeling (Bezier, B-splines)
- CAD and Engineering: Car Design, Flight Design, Ship Design, etc
- Image Interpolation: Smoothly fill missing pixels
- Data Science: Data smooth to avoid noise in datasets
- Data Signal Processing: Filter signals by smoothing out noise
- Medical Imaging: Smoother surface reconstruction from CT/MRI scan data
- Animation and Robotics: Smooth Motion Control
- Climate Modeling: Fitting Weather Data

Cubic Spline Interpolation: Advantages

- Smoothness: Produces continuous curves with smooth derivatives and avoids sharp corners
- Flexible: Closely approximate complex data sets with high degree of accuracy
- Local Control: Modify as per individual data points only affect the nearby segment.

Linear Spline

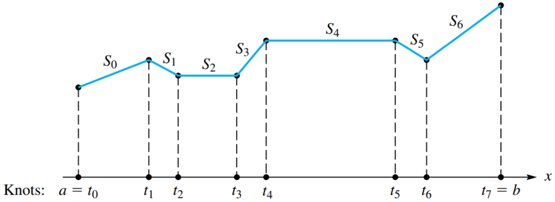
Definition 1 (Spline of Degree 1)

A function S is called a spline of degree 1 or a linear spline if

- The domain of S is an interval $[a, b]$
- S is continous on $[a, b]$
- There is a partitioning of the interval $a = t_0 < t_1 < \cdots < t_n = b$ such that S is a linear polynomial on each subinterval $[t_i, t_{i+1}]$

Linear Spline

- A **spline function** is a function that consists of polynomial pieces joined together with certain conditions.
- Polynomial function whose pieces are linear polynimial joined together to achieve continuity
- Consider the points $t_0, t_1, t_2, \cdots, t_n$. The function changes its character at t_i's are called **knots**



Linear Spline: Explicit Form

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_i(x) & x \in [x_{i-1}, x_i] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

where

$$S_i(x) = a_i x + b_i$$

Linear Spline

- Each piece of $S(x)$ is a linear polynomial.
- If the knots $x_0, x_1, \cdots, x_n$ were given and if the coefficients $a_0, b_0, a_1, b_1, \cdots, a_{n-1}, b_{n-1}$ were all known, then the evaluation of $S(x)$ at a specific x would proceed by first determining the interval that contains x and then using appropriate linear function for that interval.
- If the function S is continuous, we call it a first-degree spline.
- Outside the interval $[a, b]$, $S(x)$ is usually defined to be the same function on the left of a as it is on the leftmost interval $[x_0, x_1]$ and the same on the right of b as it is on the rightmost subinterval $[x_{n-1}, x_n]$

$$S(x) = \begin{cases} S_0(x) & x < a \\ S_{n-1}(x) & x > b \end{cases}$$

Linear Spline

- Continuity of a function f at a point s can be defined by the condition

$$\lim_{x \to s^+} f(x) = \lim_{x \to s^-} f(x) = f(s)$$

Example 2

Check whether the following $S(x)$ is a first degree spline.

$$S(x) = \begin{cases} x & x \in [-1, 0] \\ 1 - x & x \in (0, 1) \\ 2x - 2 & x \in [1, 2] \end{cases}$$

It is linear but not spline as it is discontinuous at 0.

Linear Spline Interpolation

Linear Spline Interpolation

The linear spline can be used for interpolation

- Consider $(x_0, f(x_0)), (x_1, f(x_1)), \cdots, (x_n, f(x_n))$
- We can draw a polygonal line through these points without drawing a vertical segment. This polygonal line is a graph of a function.

$$S_i(x) = f(x_i) + m_i(x - x_i), \quad x \in [x_i, x_{i+1}]$$

where m_i is the slope of the line and is therefore given by formula

$$m_i = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

Linear Spline Interpolation

Observations:

- The function S has $2n$ parameters: n coefficients a_i and n constants b_i
- We are imposing $2n$ conditions. Each constituted function S_i must interpolate the data at the ends of its subinterval.
- number of parameters = number of conditions
- For higher-degree splines, there will be a mismatch.
- The above equation is much better for practical evaluation.

Linear Spline

Example 3

Fit the following data with linear spline and evaluate the function at $x = 5$

x	3.0	4.5	7.0	9.0
$f(x)$	2.5	1.0	2.5	0.5

For fitting the data, we need to find all linear splines. However, for evaluating the function at $x = 5$, it is enough to compute the spline between 4.5 and 7.5, that is,

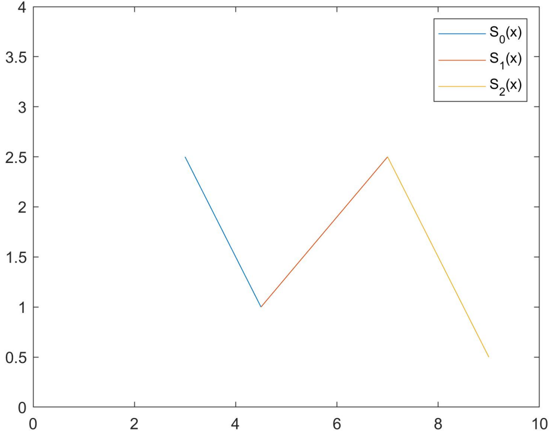
$$S_1(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1), x \in [x_1, x_2]$$

That is,

$$S_1(x) = f(4.5) + \frac{2.5 - 1}{7 - 4.5}(x - 4.5) = 1 + 0.60(x - 4.5)$$

Hence $S_1(5) = f(5) = 1.3$

Linear Spline



Theorems on Linear Spline

Modulus of Continuity

In order to find the goodness of fit for a linear spline, let us introduce modulus of continuity.

Definition 4 (Modulus of Continuity)

Let f be a function defined on the interval $[a, b]$. The modulus of continuity of f is

$$\omega(f; h) = \sup\{|f(u) - f(v)| : a \leq u \leq v \leq b, |u - v| \leq h\}$$

- The quantity $\omega(f; h)$ measures how much f can change over a small interval of width h.
- If f is continuous, then f is uniformly continuous and ω will tend to zero as h tends to zero.
- If f is not continuous, ω will not tend to zero.

Modulus of Continuity

- If f is differentiable, and if f' is bounded on (a, b), then by MVT, we can obtain the estimate of ω.

$$|f(u) - f(v)| = |f'(c)(u - v)| \leq M_1|u - v| \leq M_1 h$$

- If $f(x) = x^3$ on $[1, 4]$, then $\omega \leq 48h$

First-degree Polynomial Accuracy Theorem

Theorem 5 (First-degree Polynomial Accuracy Theorem)
If p is the first-degree polynomial that interpolates a function f at the endpoints of an interval $[a, b]$, then with $h = b - a$, we have

$$|f(x) - p(x)| \leq \omega(f; h) \quad x \in [a, b]$$

Proof: The linear function p is given explicitly by the formula,

$$p(x) = \left(\frac{x-a}{b-a}\right) f(b) + \left(\frac{b-x}{b-a}\right) f(a)$$

$$\implies f(x) - p(x) = \left(\frac{x-a}{b-a}\right) [f(x) - f(b)] + \left(\frac{b-x}{b-a}\right) [f(x) - f(a)]$$

First-degree Polynomial Accuracy Theorem

Proof (Continued):

$$
\begin{aligned}
|f(x) - p(x)| &\leq \left(\frac{x-a}{b-a}\right)|f(x) - f(b)| + \left(\frac{b-x}{b-a}\right)|f(x) - f(a)| \\
&\leq \left(\frac{x-a}{b-a}\right)\omega(f;h) + \left(\frac{b-x}{b-a}\right)\omega(f;h) \\
&= \left(\frac{x-a}{b-a} + \frac{b-x}{b-a}\right)\omega(f;h) \\
&= \omega(f;h)
\end{aligned}
$$

Linear Spline Accuracy Theorem

Theorem 6 (Linear Spline Accuracy Theorem)
If p is a linear spline having knots $a = x_0 < x_1 < \cdots < x_n = b$. If p interpolates a function f at these knots, then with $h = \max_i (x_i - x_{i-1})$, we have

$$|f(x) - p(x)| \leq \omega(f; h) \quad x \in [a, b]$$

Proof: Apply the above theorem for each subinterval.

- The first theorem tells us that if more knots are inserted in such a way that the maximum spacing h goes to zero, then the corresponding linear spline will converge to f.

Quadratic Splines

Quadratic Splines

- Splines of degree higher than 1 are more complicated
- Let us use Q to remind ourselves that we are considering piecewise quadratic functions.

Definition 7 (Quadratic Splines)

A function Q is called quadratic splines if

- The domain of Q is an interval $[a, b]$
- Q and Q' are continuous on $[a, b]$
- There are points x_i (called knots) such that $a = x_0 < x_1 < \cdots < x_n = b$ and Q is a polynomial of degree at most 2 on each subinterval $[x_i, x_{i+1}]$

Quadratic Spline

Example 8

Check whether the following $S(x)$ is a first degree spline.

$$Q(x) = \begin{cases} x^2 & x \in [-10, 0] \\ -x^2 & x \in (0, 1) \\ 1 - 2x & x \in [1, 20] \end{cases}$$

It is a quadratic spline, for,

$$\begin{aligned}
\lim_{x\to 0^-} Q(x) &= \lim_{x\to 0^-} x^2 &&= 0, & \lim_{x\to 0^+} Q(x) &= \lim_{x\to 0^-} (-x^2) &&= 0 \\
\lim_{x\to 1^-} Q(x) &= \lim_{x\to 1^+} (-x^2) &&= -1, & \lim_{x\to 1^+} Q(x) &= \lim_{x\to 0^-} (1-2x) &&= -1 \\
\lim_{x\to 0^-} Q'(x) &= \lim_{x\to 0^-} 2x &&= 0, & \lim_{x\to 0^+} Q'(x) &= \lim_{x\to 0^+} (-2x) &&= 0 \\
\lim_{x\to 1^-} Q'(x) &= \lim_{x\to 1^-} (-2x) &&= -2, & \lim_{x\to 1^+} Q'(x) &= \lim_{x\to 1^+} (-2) &&= -2
\end{aligned}$$

Quadratic Splines

- The objective of the quadratic splines is to derive second-order polynomial for each interval between data points. That is,

$$Q_i(x) = a_i x^2 + b_i x + c_i$$

- For $n + 1$ data points, there are n intervals and $3n$ unknown constants to evaluate.
- Requirement: $3n$ conditions to evaluate unknowns

Quadratic Splines

- The function values of adjacent polynomials must be equal to the interior knots.

$$Q_{i-1}(x_{i-1}) = f(x_{i-1}) \implies a_{i-1}x_{i-1}^2 + b_{i-1}x_{i-1} + c_{i-1} = f(x_{i-1}) \quad (1)$$

$$Q_i(x_{i-1}) = f(x_{i-1}) \implies a_i x_{i-1}^2 + b_i x_{i-1} + c_i = f(x_{i-1}) \quad (2)$$

for $i = 2, 3, \cdots, n$

- Since only interior knots are used, we obtain $2n - 2$ conditions
- The first and last functions must pass through the end points.

$$a_1 x_0^2 + b_1 x_0 + c_1 = f(x_0) \quad (3)$$

$$a_n x_n^2 + b_n x_n + c_n = f(x_n) \quad (4)$$

- Hence a total of $2n$ conditions.

Quadratic Splines

- The first derivatives at the interior knots must be equal

$$Q'_{i-1}(x_{i-1}) = Q'_i(x_{i-1}) \implies 2a_{i-1}x_{i-1} + b_{i-1} = 2a_i x_{i-1} + b_i \qquad (5)$$

  for $i = 2, 3, \cdots, n$
- It provides another $n-1$ conditions
- Hence, we have $3n-1$ conditions and $3n$ unknowns. Therefore, there is a shortfall of 1 condition. There are different ways to obtain the condition, but let us use the following:
- Assume that second derivative is zero at the first point. That is,

$$a_1 = 0 \qquad (6)$$

- Hence a total of $3n$ conditions.

Quadratic Spline

Example 9

Fit the following data with quadratic spline and evaluate the function at $x = 5$

x	3.0	4.5	7.0	9.0
$f(x)$	2.5	1.0	2.5	0.5

For this case, we need to find $3n$ unknowns, here $n = 3$, therefore, 9 unknowns must be determined

$$Q_{i-1}(x_{i-1}) = f(x_{i-1}) \implies a_{i-1}x_{i-1}^2 + b_{i-1}x_{i-1} + c_{i-1} = f(x_{i-1})$$
$$Q_i(x_{i-1}) = f(x_{i-1}) \implies a_i x_{i-1}^2 + b_i x_{i-1} + c_i = f(x_{i-1})$$

for $i = 2, 3$

Quadratic Spline

Using equations (1) and (2), we obtain

$$
\begin{aligned}
4.5^2 a_1 + 4.5 b_1 + c_1 &= 1.0 \\
4.5^2 a_2 + 4.5 b_2 + c_2 &= 1.0 \\
7^2 a_2 + 7 b_2 + c_2 &= 2.5 \\
7^2 a_3 + 7 b_3 + c_3 &= 2.5
\end{aligned}
$$

From equations (3) and (4), we obtain

$$
\begin{aligned}
3^2 a_1 + 3 b_1 + c_1 &= 2.5 \\
9^2 a_3 + 9 b_3 + c_3 &= 0.5
\end{aligned}
$$

Quadratic Spline

Using equations (5), we obtain

$$9a_1 + b_1 = 9a_2 + b_2$$
$$14a_2 + b_2 = 14a_3 + b_3$$

Finally, we have

$$a_1 = 0$$

Quadratic Spline

Solving for unknowns, we obtain

$$\begin{pmatrix} 4.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20.25 & 4.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 49 & 7 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 49 & 7 & 1 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 81 & 9 & 1 \\ 1 & 0 & -9 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 14 & 1 & 0 & -14 & -1 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2.5 \\ 2.5 \\ 2.5 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$$

Quadratic Spline

Upon solving this we obtain

$$\begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 5.5 \\ 0.64 \\ -6.76 \\ 18.46 \\ -1.6 \\ 24.6 \\ -91.3 \end{pmatrix}$$

Quadratic Spline

Hence the Quadratic spline is given by

$$Q(x) = \begin{cases} -x + 5.5 & x \in [3, 4.5] \\ 0.64x^2 - 6.76x + 18.46 & x \in [4.5, 7.0] \\ -1.6x^2 + 24.6x - 91.3 & x \in [7, 9] \end{cases}$$

When we use f_2 and $Q(5) = f(5) = f_2(5) = 0.66$

Quadratic Spline

A Quadratic spline consists of n separate quadratic functions $x \to a_i x^2 + b_i x + c_i$ one for each subinterval created by the $n+1$ knots.

$$Q(x) = \begin{cases} Q_0(x) & x \in [x_0, x_1] \\ Q_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ Q_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

It is continuously differentiable on the entire interval $[x_0, x_n]$, that is $Q(x_i) = f(x_i)$.
Since Q' is continuous, let $z_i = Q'(x_i)$

Quadratic Spline

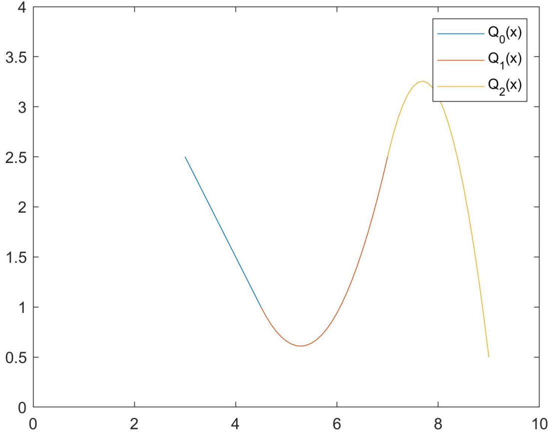
The following formula for Q_i is given as below

$$Q_i(x) = \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2 + z_i(x - x_i) + f(x_i) \tag{7}$$

We can verify that $Q_i(x_i) = f(x_i)$ and $Q_i'(x_i) = z_i$. Also, $Q_i'(x_{i+1}) = z_{i+1}$ where

$$z_{i+1} = -z_i + 2\left(\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}\right) \tag{8}$$

Quadratic Spline



Cubic Splines

Spline of degree k

Definition 10 (Spline of degree k)
A function S is called a spline of degree k if

- The domain of S is an interval $[a, b]$
- $S, S', S'', \cdots, S^{(k-1)}$ are continuous on $[a, b]$
- There are points x_i (called knots) such that $a = x_0 < x_1 < \cdots < x_n = b$ and S is a polynomial of degree at most k on each subinterval $[x_i, x_{i+1}]$

Cubic Spline

Definition 11 (Cubic Spline)

A function C is called a cubic spline if

- The domain of C is an interval $[a, b]$
- C, C', C'' are continuous on $[a, b]$
- There are points x_i (called knots) such that $a = x_0 < x_1 < \cdots < x_n = b$ and C is a polynomial of degree at most 3 on each subinterval $[x_i, x_{i+1}]$

Cubic Spline

A cubic spline consists of n separate cubic functions $x \to a_i x^3 + b_i x^2 + c_i x + d_i$ one for each subinterval created by the $n+1$ knots.

$$C(x) = \begin{cases} C_0(x) & x \in [x_0, x_1] \\ C_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ C_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

There are $4n$ unknowns, hence, we require $4n$ conditions.

Cubic Spline

We can use the following $4n$ conditions

- The function values must be equal at the interior knots ($2n - 2$ conditions)
- The first and last function must pass through the end points (2 conditions)
- The first derivatives at the interior knots must be equal ($n - 1$ conditions)
- The second derivatives at the interior knots must be equal ($n - 1$ conditions)
- The second derivatives at the end knots are zero (2 conditions)

Cubic Spline

Cubic splines for each interval is given by

$$\begin{aligned} C_i(x) = {} & \frac{C_i''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{C_i''(x_i)}{6(x - x_{i-1})}(x - x_{i-1})^3 \\ & + \left[\frac{f(x_{i-1})}{x_i - x_{i-1}} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x) \\ & + \left[\frac{f(x_i)}{x_i - x_{i-1}} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1}) \end{aligned} \tag{9}$$

This equation contains only two unknowns-the second derivatives at the end of each interval.

Cubic Spline

These unknowns can be evaluated using the following equation

$$
\begin{aligned}
&(x_i - x_{i-1})C''(x_{i-1}) + 2(x_{i+1} - x_{i-1})C''(x_i) + (x_{i+1} - x_i)C''(x_{i+1}) \\
&\quad = \frac{6}{x_{i+1} - x_i}[f(x_{i+1}) - f(x_i)] + \frac{6}{x_i - x_{i-1}}[f(x_{i-1}) - f(x_i)]
\end{aligned} \tag{10}
$$

If this equation is written for all interior knots $n-1$ simultaneous equations result with $n-1$ unknowns.

Cubic Spline

Example 12

Fit the following data with cubic spline and evaluate the function at $x = 5$

x	3.0	4.5	7.0	9.0
$f(x)$	2.5	1.0	2.5	0.5

For this case, we need to find $4n$ unknowns, here $n = 3$, therefore, 12 unknowns must be determined

By equation (10), we get

$$(4.5-3)C''(3) + 2(7-3)C''(4.5) + (7-4.5)C''(7)$$
$$= \frac{6}{7-4.5}[2.5-1] + \frac{6}{4.5-3}[2.5-1]$$

Cubic Spline

Since $C''(3) = 0$, we obtain,

$$8C''(4.5) + 2.5C''(7) = 9.6$$

Similarly, we obtain

$$2.5C''(4.5) + 9C''(7) = -9.6$$

Upon solving, we get

$$C''(4.5) = 1.67909, C''(7) = -1.53308$$

Cubic Spline

Now

$$C_i(x) = \frac{C_i''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{C_i''(x_i)}{6(x - x_{i-1})}(x - x_{i-1})^3$$
$$+ \left[\frac{f(x_{i-1})}{x_i - x_{i-1}} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x)$$
$$+ \left[\frac{f(x_i)}{x_i - x_{i-1}} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$
$$C_1(x) = \frac{1.67909}{6(4.5 - 3)}(x - 3)^3 + \left[\frac{2.5}{4.5 - 3}\right](4.5 - x)$$
$$+ \left[\frac{1}{4.5 - 3} - \frac{1.67909(4.5 - 3)}{6}\right](x - 3)$$

Cubic Spline

Hence,

$$C_0(x) = 0.186566(x-3)^3 + 1.666667(4.5-x) + 0.246894(x-3)$$

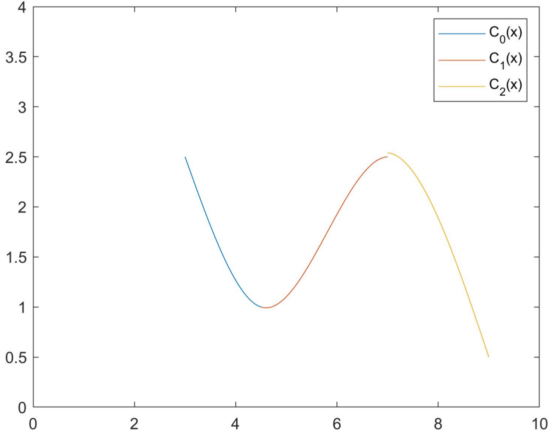
Similarly,

$$C_1(x) = 0.111939(7-x)^3 - 0.102205(x-4.5)^3 - 0.299621(7-x) + 1.638783(x-4.5)$$

$$C_2(x) = -0.127757(9-x)^3 + 1.761027(9-x) + 0.25(x-7)$$

Hence $C_2(5) = 1.102886$

Cubic Spline



Comparison

Property	Linear	Quadratic	Cubic
Continuity	$C[a,b]$	$C^1[a,b]$	$C^2[a,b]$
Smoothness	No (kinks)	First derivative smooth	Very smooth no kinks
Computation	Low	Moderate	Higher
Accuracy	Low	Medium	High
Application	Simple Models	Application with first derivative smooth is sufficient	Physics, Graphics, data fitting, medical imaging

Comparison

Feature	Polynomial	Spline
Definition	Single high-degree polynomial to fit all data points	Piecewise low-degree polynomials
Smoothness	C^∞	C^2 for cubic
Oscillation	Oscillate	No large oscillations
Computation	Requires large solving system	Tridiagonal system solver
Accuracy	High for small datasets but unstable for large datasets	High accuracy but stable for large datasets
Runge's phenomenon	Severe oscillations for for large datasets	No oscillations stable for large datasets
Applications	Good for small datasets	Good for large datasets smooth and stable approximations

Bezier Curve

Parametric Curves

Linear Bezier Curve

$$B(t) = (1-t)P_0 + tP_1, 0 \leq t \leq 1$$

Quadratic Bezier Curve

$$B(t) = (1-t)^2 P_0 + 2(1-t)tP_1 + t^2 P_2, 0 \leq t \leq 1$$

Cubic Bezier Curve

$$B(t) = (1-t)^3 P_0 + 3(1-t)^2 tP_1 + 3(1-t)t^2 P_2 + t^3 P_3, 0 \leq t \leq 1$$

Parametric Curves

$$B(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i P_i$$

Thanks

Doubts and Suggestions

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MA633L-Numerical Analysis

Lecture 14 : Spline Interpolation

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