**INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI**
**DEPARTMENT OF MATHEMATICS AND STATISTICS**

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# MA635P-Scientific Programming Laboratory

Lab Exercise-2 (180 Marks) Deadline: 16 January 2025, 5:00 PM

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## Exercise 1 : Taylor Series

$$S_1(x) = \sum_{i=0}^{n-1} (-1)^i \frac{x^{2i+1}}{(2i+1)!} \quad \text{for } \sin x, \qquad S_2(x) = \sum_{i=0}^{n-1} (-1)^i \frac{x^{2i}}{(2i)!} \quad \text{for } \cos x$$

$$S_3(x) = \sum_{i=0}^{n-1} \frac{x^i}{i!} \quad \text{for } e^x, \qquad S_4(x) = \sum_{i=1}^{n} (-1)^{i+1} \frac{x^{2i-1}}{2i-1} \quad \text{for } \tan^{-1} x$$

1. Let $T_5(n,x)$, $T_6(n,x)$, $T_7(n,x)$ and $T_8(n,x)$ be the number of elementary operations (float/double additions and multiplications only) involved in $S_1(x)$, $S_2(x)$, $S_3(x)$ and $S_4(x)$ respectively. Fill the table 1 and 2
2. **Paper Work:** Compute the general $T_5(n,x), T_6(n,x), T_7(n,x), T_8(n,x)$. [40]

| $x$ | $n$ | $T_5(n,x)$ | Seconds | $T_6(n,x)$ | Seconds |
|---|---|---|---|---|---|
| $\pi/3$ | 10 | | | | |
| $\pi/4$ | 20 | | | | |
| $\pi/6$ | 40 | | | | |

Tab. 1: $\sin(x)$ and $\cos(x)$

| $x$ | $n$ | $T_7(n,x)$ | Seconds | $T_8(n,x)$ | Seconds |
|---|---|---|---|---|---|
| 0 | 10 | | | | |
| 1 | 20 | | | | |
| $-1$ | 30 | | | | |

Tab. 2: $\exp(x)$ and $\tan^{-1}(x)$

## Exercise 2: Big O

Suppose an algorithm takes 1 minute to compute 10 operations or to process 10 data. How many operations/data will be processed in one hour (H), 1 day (D), 1 week (W), 1 year (Y), 1 decade (D) and 1 century (C) by the algorithm if $T(n)$ takes one of the following form in the table 3? Write a Python/C++ program to fill the following table 3. [40]

| $T(n)$ | **Name** | c | **1 m** | **1 H** | **1 D** | **1 W** | **1Y** | **10 Y** | **1 C** |
|---|---|---|---|---|---|---|---|---|---|
| $O(\log(n))$ | logarithmic | | 10 | | | | | | |
| $O((\log^2(n)))$ | poly-logarithmic | | 10 | | | | | | |
| $O(n)$ | linear | | 10 | | | | | | |
| $O(n^2)$ | quadratic | | 10 | | | | | | |
| $O(n^3)$ | cubic | | 10 | | | | | | |
| $O(n^{1.5})$ | polynomial | | 10 | | | | | | |
| $O(2^n)$ | exponential | | 10 | | | | | | |

Tab. 3: Big O

**Hint:** For $O(n)$: $T(n) = cn \implies T(10) = 60 \implies 60 = 10c \implies c = 6 \implies T(n) = 6n$
For 1 Hour:

$$T(n) = 3600 \implies 6n = 3600 \implies n = 600$$

1 Minute: 10, 1 Hour: 600, 1 Day: 14400, 1 Week: 100800, 1 Year: $5.26 \times 10^6$, 10 Years: $5.26 \times 10^7$, 100 Years: $5.26 \times 10^8$

For $O(n^2)$: $T(n) = cn^2 \implies T(10) = 60 \implies 60 = 100c \implies c = 0.6 \implies T(n) = 0.6n^2$
For 1 Hour:

$$T(n) = 3600 \implies 0.6n^2 = 3600 \implies n = 77$$

1 Minute: 10, 1 Hour: 77

# Exercise 3: File Size and Vector Operations Count

1. Use random library to create a random vector $X$ of size 5 and another vector $Y$ of size 5.
2. Store/Write the vector in a file VectorX.txt and VectorY.txt as given in below format?

   **Vector.txt**
   5
   0.1575
   0.7924
   0.9845
   0.2563
   0.4866

3. What is the size of the file VectorX.txt or VectorY.txt?
4. Let $S(n, X)$ denote the size of the file VectorX.txt. What will be the size of the file VectorX.txt if $n$ is as given in the following table 4?
5. Let $T_+(n, X, Y)$ denote the number of additions involved while adding two vectors. Fill the following table 4.

6. Let $T_+^*(n, X, Y)$ denote the number of multiplications and additions involved while computing dot product of two vectors. Fill the following table 4. [25]

| $n$ | $S(n, X)$ | $T_+(n, X, Y)$ | $T_+^*(n, X, Y)$ |
|---|---|---|---|
| 10 | | | |
| 100 | | | |
| 1000 | | | |
| $10^6$ | | | |
| $10^9$ | | | |

Tab. 4: Vector Storage and Operations Count

# Exercise 4: Matrix Operations and Matrix-Vector Product Count

1. Use random library to create a random matrices $A$ and $B$ of size $5 \times 5$
2. Store/Write the matrix in a file MatrixA.txt and MatrixB.txt as below

   **MatrIx.txt**
   5,5
   0.8147,0.0975,0.1576,0.1419,0.6557
   0.9058,0.2785,0.9706,0.4218,0.0357
   0.1270,0.5469,0.9572,0.9157,0.8491
   0.9134,0.9575,0.4854,0.7922,0.9340
   0.6324,0.9649,0.8003,0.9595,0.6787

3. What is the size of the file MatrixA.txt or MatrixB.txt?
4. Let $S(n, A)$ denote the size of the file MatrixA.txt. What will be the size of the file MatrixA.txt if $n$ is as given in the table 5?
5. Let $T_+(n, A, B)$ denote the number of additions involved while adding two square matrices. Fill the table 5.
6. Let $T_+^*(n, A, B)$ denote the number of multiplications and additions involved while multiplying two square matrices. Fill the table 5.
7. Let $T_+^*(n, A, X)$ denote the number of multiplications and additions involved for Matrix Vector Multiplications. Fill the table 5. $[7 \times 7 = 49 + 1]$

| $n$ | $S(n,A)$ | $T_+(n,A,B)$ | $T_+^*(n,A,B)$ | $T_+^*(n,A,X)$ |
|---|---|---|---|---|
| 10 | | | | |
| 100 | | | | |
| 1000 | | | | |
| $10^6$ | | | | |
| $10^9$ | | | | |

Tab. 5: Matrix Storage and Operations Count

# Exercise 5: Strassen algorithm

Let $A$ and $B$ matrices of size $2^n \times 2^n$. To compute matrix-matrix product $C = AB$, split matrices into equally sized block matrices:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

where $A_{ij}, B_{ij}$ and $C_{ij}$ are of size $2^{n-1} \times 2^{n-1}$. Compute

- $M_1 = (A_{11} + A_{22})(B_{11} + B_{22})$
- $M_2 = (A_{21} + A_{22})B_{11}$
- $M_3 = A_{11}(B_{12} - B_{22})$
- $M_4 = A_{22}(B_{21} - B_{11})$
- $M_5 = (A_{11} + A_{12})B_2 2$
- $M_6 = (A_{21} - A_{11})(B_{11} + B_{12})$
- $M_7 = (A_{12} - A_{22})(B_{21} + B_{22})$
- $C_{11} = M_1 + M_4 - M_5 + M_7$
- $C_{12} = M_3 + M_5$
- $C_{21} = M_2 + M_1$
- $C_{22} = M_1 - M_2 + M_3 + M_6$

1. Let $ST_+^*(n,A,B)$ denote the number of multiplications and additions involved while multiplying two square matrices. Fill the table 6 using Strassen Algorithm. Here $T_+^*$ is same as Exercise 4. $[6 \times 4 = 24 + 1]$

| $n$ | $S(n,A)$ | $ST_+^*(n,A,B)$ | $T_+^*(n,A,B)$ |
|---|---|---|---|
| 4 | | | |
| 16 | | | |
| 256 | | | |
| 512 | | | |
| 1024 | | | |
| 2048 | | | |

Tab. 6: Strassen Algorithm Operations Count