MA635P-Scientific Programming Laboratory

Radial Basis Function Interpolation

Panchatcharam Mariappan[1]

[1]Associate Professor
Department of Mathematics and Statistics
IIT Tirupati, Tirupati

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भारतीय प्रौद्योगिकी संस्थान तिरुपति

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Radial Basis Function (RBF) Interpolation

Background

- Radial Basis Function (RBF) interpolation is a powerful meshfree method for approximating multivariate functions from scattered data.
- Unlike polynomial interpolation, RBF methods work naturally in higher dimensions.
- Widely used in geospatial modeling, fluid simulations, machine learning, and PDE discretization.

Mathematical Formulation

Given scattered data points (x_i, f_i), the RBF interpolant is:

$$S(x) = \sum_{i=1}^{N} \lambda_i \, \phi(\|x - x_i\|)$$

where:

- $\phi(r)$ is a radial basis function.
- Coefficients λ_i are obtained from:

$$S(x_j) = f_j$$

Common Radial Basis Functions

- Gaussian: $\phi(r) = e^{-(\varepsilon r)^2}$
- Multiquadric: $\phi(r) = \sqrt{1 + (\varepsilon r)^2}$
- Inverse Multiquadric: $\phi(r) = \frac{1}{\sqrt{1+(\varepsilon r)^2}}$
- Thin Plate Spline: $\phi(r) = r^2 \log r$

Parameter ε is called the shape parameter.

Objective

- To implement RBF interpolation for scattered multivariate data.
- To analyze stability and conditioning of the interpolation matrix.
- To study the effect of the shape parameter ε.
- To compare RBF interpolation with Shepard and Polynomial interpolation.

Requirements

Read the following papers/Lecture Notes:
1. Meshfree Approximation Methods with MATLAB.
2. Radial Basis Function: Theory and Implementations

Steps to Solve

1. Code
   1.1 Python implementation of Gaussian and Multiquadric RBF.
   1.2 Solve linear system for λ_i.
2. Mathematical Report
   2.1 Derivation of interpolation matrix.
   2.2 Conditions for invertibility.
3. Error Analysis
   3.1 Study convergence with increasing N.
   3.2 Analyze effect of ε.

Advanced Investigation

1. Conditioning Study
   1.1 Analyze ill-conditioning for small ε.
2. Convergence in Terms of Fill Distance

$$h = \sup_{x \in \Omega} \min_i \| x - x_i \|$$

Study empirical convergence rate.

Applications

1. Geospatial Surface Reconstruction.
2. Image Reconstruction from scattered pixels.

Deliverable

1. Python code implementation.
2. A report including:
   - Mathematical derivation
   - Stability and conditioning study
   - Error analysis
   - Image Reconstruction from scattered pixels.
3. Numerical experiments and visualizations.

Thanks

Doubts and Suggestions

panch.m@iittp.ac.in

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