MA635P-Scientific Programming Laboratory

B-Spline Interpolation

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B-Spline Interpolation

Background

- B-Splines (Basis Splines) are piecewise polynomial functions defined over a knot vector.
- They provide local support, numerical stability, and smoothness control.
- Widely used in computer graphics, CAD, isogeometric analysis, and numerical PDEs.

Mathematical Formulation

Given a knot vector:

$$t_0 \leq t_1 \leq \cdots \leq t_{n+p+1}$$

The B-spline basis functions of degree p are defined recursively:

$$N_{i,0}(x) = \begin{cases} 1 & t_i \leq x < t_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(x) = \frac{x - t_i}{t_{i+p} - t_i} N_{i,p-1}(x) + \frac{t_{i+p+1} - x}{t_{i+p+1} - t_{i+1}} N_{i+1,p-1}(x)$$

Spline Interpolant

The spline interpolant is:

$$S(x) = \sum_{i=0}^{n} c_i N_{i,p}(x)$$

where coefficients c_i are determined from:

$$S(x_j) = f(x_j)$$

Key Properties:

- Local support
- Partition of unity
- C^{p-1} continuity (for simple knots)

Objective

- To implement B-spline interpolation for 1D data.
- To analyze smoothness and stability properties.
- To compare B-splines with polynomial interpolation.
- To study effect of knot placement.

Requirements

Read the following references:

1. A Practical Guide to Splines
2. The NURBS Book

Steps to Solve

1. Code
   - 1.1 Implement Cox–de Boor recursion formula.
   - 1.2 Construct spline interpolation matrix.
2. Mathematical Report
   - 2.1 Derivation of B-spline basis.
   - 2.2 Proof of partition of unity.
   - 2.3 Continuity analysis.
3. Error Analysis
   - 3.1 Study convergence as number of knots increases.
   - 3.2 Compare with global polynomial interpolation.

Advanced Investigation

1. Knot Placement Study
   - 1.1 Uniform vs adaptive knot distribution.
   - 1.2 Effect on interpolation error.
2. Conditioning Analysis
   - 2.1 Compute condition number of spline interpolation matrix.
   - 2.2 Compare with high-degree polynomial interpolation.
3. Extension to 2D Tensor-Product B-Splines

$$S(x, y) = \sum_{i,j} c_{ij} N_{i,p}(x) N_{j,q}(y)$$

Study surface reconstruction.

Applications

1. Curve and surface modeling in CAD.
2. Isogeometric Analysis for PDEs.
3. Data smoothing and signal reconstruction.
4. Path planning and trajectory generation.

Deliverable

1. Python implementation of B-spline interpolation.
2. A report including:
    - Mathematical derivation
    - Smoothness and stability analysis
    - Error comparison with polynomial interpolation
    - Numerical surface reconstruction example
3. Visualization of basis functions and spline curves.

Thanks

Doubts and Suggestions

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