

MA635P-Scientific Programming Laboratory

Sparse Grid Interpolation (Smolyak Algorithm)

Panchatcharam Mariappan¹

¹Associate Professor
Department of Mathematics and Statistics
IIT Tirupati, Tirupati

Deadline: March 16, 2026





Sparse Grid Interpolation (Smolyak Algorithm)

Background



- High-dimensional interpolation suffers from the curse of dimensionality.
- Tensor-product grids require $O(N^d)$ points in d dimensions.
- The Smolyak algorithm constructs sparse grids reducing complexity significantly.
- Widely used in uncertainty quantification, PDEs, finance, and high-dimensional integration.

Curse of Dimensionality



For tensor-product interpolation:

$$\text{Total points} = n^d$$

Example:

- $n = 20, d = 6$
- Total points = $20^6 = 64,000,000$

Sparse grids reduce this to approximately:

$$O(n(\log n)^{d-1})$$

Smolyak Construction



The Smolyak interpolation operator is:

$$\mathcal{A}(q, d) = \sum_{|\mathbf{i}| \leq q+d-1} (\Delta_{i_1} \otimes \cdots \otimes \Delta_{i_d})$$

where:

- Δ_i are 1D incremental interpolation operators.
- $\mathbf{i} = (i_1, \dots, i_d)$
- q controls accuracy level.

Objective

- To implement 1D hierarchical interpolation.
- To construct sparse grid interpolation using Smolyak algorithm.
- To compare sparse grid with full tensor-product interpolation.
- To analyze computational complexity and accuracy.



Requirements

Read the following references:

1. Smolyak quadrature formulas to the numerical integration
2. Smolyak Method for Solving Dynamic Economic Models



Steps to Solve



1. Code
 - 1.1 Implement hierarchical 1D interpolation.
 - 1.2 Construct Smolyak sparse grid in 2D and 3D.
2. Mathematical Report
 - 2.1 Derivation of Smolyak formula.
 - 2.2 Complexity analysis.
3. Error Analysis
 - 3.1 Compare with Lagrange interpolation.
 - 3.2 Study convergence for smooth test functions.

Advanced Investigation



1. Convergence in Sobolev Spaces

$$\|f - \mathcal{A}(q, d)\| \leq CN^{-k}(\log N)^{(d-1)(k+1)}$$

Study empirical convergence rates.

2. Dimension-Adaptive Sparse Grids

2.1 Implement adaptive refinement strategy.

2.2 Identify anisotropic functions.

Applications

1. High-dimensional PDE approximation.
2. Uncertainty Quantification (UQ).
3. Financial option pricing.
4. Scientific Machine Learning (surrogate modeling).



Deliverable



1. Python implementation of sparse grid interpolation.
2. A report including:
 - Mathematical derivation of Smolyak formula
 - Complexity analysis
 - Error comparison with tensor-product interpolation
 - Numerical experiments in 2D and 3D
3. Visualization of sparse vs full grid structure.

Thanks

Doubts and Suggestions

panch.m@iittp.ac.in



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